



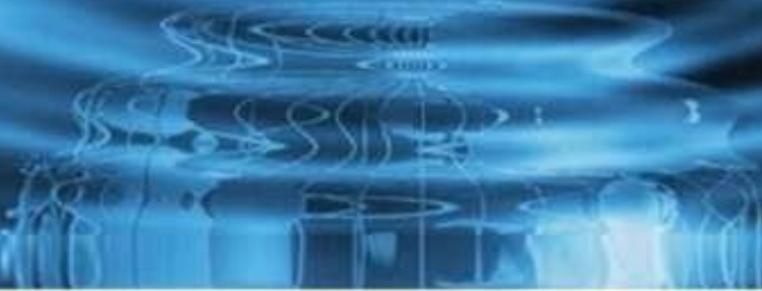
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Multi-Objective Transportation Problems in Fuzzy Environments Solved by Combining the Geometric Mean Method with the Ant Colony Optimization Algorithm

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Abstract

An optimisation hotspot and a common headache for entrepreneurs everywhere is the transportation issue (TP). Saving money on transportation is the main objective when it comes to getting materials from where they are to where they need to go. The literature shows that many methods have been established with just one objective in mind, even if TPs aren't always made with two objectives in mind. Addressing transportation challenges while juggling many goals is a typical undertaking. An innovative geometric approach to solving multi-criteria TP and a fresh take on the Ant Colony Optimisation algorithm (ACO) for solving multi-objective TP in a fuzzy setting are both highlighted in this work. Several fields have found practical applications for fuzzy numbers, such as optimisation and operations research. For quite some time, the ACO Algorithm has been known as an alternate approach to optimisation issues. This research aims to improve the ACO algorithm for solving the Multi-Objective TP model and to provide a novel method for organising fuzzy numbers. According to the objective values, our technique, which is called Geometric Mean Ant Colony Optimisation Algorithm (GMACOA), is superior to other methods. To show how the approach compares to other methods that are currently in use, numerical examples are given.

Key Words: Multicriteria distribution problem, Ant Colony Optimization algorithm, Geometric mean, Fuzzy environment.

Introduction

In addition, TP is one of the most important distribution problems in Operations Research (OR). OR has numerous uses in engineering, business, and government systems. Daily, it is also employed to solve problems in the manufacturing and service industries. The TP is a well-known optimization problem in OR that takes a single objective function into account; nevertheless, in real-world applications, two or more criteria are more important than any single criterion. When delivering a homogeneous product from a source to a destination, the decision-maker takes numerous aspects into account, including transportation costs, a fixed price for an open route, product delivery time, deterioration rate of commodities, and so on.

The TP treats many objective functions at the same time to accommodate the criteria [1]. Proposed the transportation problem originally, while Koopmans (1949) studied it in detail in the Optimal Utilization of Transportation System. On the other side, created efficient methods for discovering solutions, and Charnels, devised the stepping stone method. Furthermore, numerous researchers are

working on this topic. In this paper, we look at various methods for solving a balanced and unbalanced transportation problem using fuzzy numbers and the ant colony algorithm [2, 3].

Fuzzy set theory has been used in a variety of domains, including OR, management science, and control theory. In real-world scenarios, supply, demand, and unit transportation costs are all uncertain [4]. Used fuzzy programming approaches to handle multi-objective linear programming issues. Several strategies for solving transportation problems in fuzzy environments are proposed in the literature, such as the concept of fuzzy set, which was introduced by in 1965. Bellman and discussed the concept of decision-making in a fuzzy environment [5, 6].

Many writers have researched fuzzy linear programming problem approaches since this pioneering work, including who demonstrated that solutions produced by fuzzy linear programming are always efficient, and among others. A fuzzy transportation problem is one in which the decision parameters are fuzzy integers. Chanas et al. studied various TP situations with interval and fuzzy param-

ters [7, 8]. The goal of the fuzzy transportation problem is to move some products from various sources to various destinations while incurring the least amount of fuzzy transportation costs and satisfying the fuzzy supply and demand requirements.

Presented a fuzzy compromised programming strategy for MOTP [9]. By giving weights to objectives, the decision maker's preferences are taken into account. Introduced a preference-based fuzzy GPA for solving a MOTP with fuzzy coefficients. They explain the fuzzy goal's membership function. This method converts membership functions into membership goals. The Euclidean distance function is utilized to provide the suitable preference structure of goals. K.B. Provided a review of the various techniques employed in MOTP. This document compiles all possible work on MOTP and provides an overview of several methodologies such as goal programming, fuzzy techniques, and evolutionary algorithms. Proposed a novel way to determine a fair MOTP solution. It is suggested in this strategy to create a sum of objectives. Patel et al (2018) proposed a new row maxima approach to solve MOTP. M. offered a product method to solve MOTP by utilizing a fuzzy membership function. Suggested a straightforward method for determining the optimal linear MOTP compromise solution. To solve MOTP, proposed the Matrix maxima approach with a Pareto optimality criterion. Used fuzzy programming to find the best compromise solution to a multi-objective transportation problem. Used the solving Multi-objective Transportation Problem by row maxima method. Proposed the Geometric Mean Method for Solving Multi-Objective Transportation Problems in Fuzzy Environments [10-18].

Furthermore, how ants can find the shortest paths between food sources and their colony. These ideas are based on ant behavior in the wild. This concept was created using the probabilistic technique known as finding good pathways via graphs. This is known as the Ant Colony Algorithm (ACA), and it was first presented by while traveling in this manner, the ants deposit a chemical compound known as a pheromone, which aids in communication among themselves. When finding the quickest path between food sources and their nest, they look for areas with high pheromone concentrations [19].

Because ants can detect pheromones and choose the most advantageous way. Dorigo, brought the concept of the ant system into the literature. The ant algorithm with elitist ants was proposed by. Following that, many writers researched ACA, including the max-min ant system, the ant algorithm with additional reinforcement, and the best-worst ant system, among others. Many optimization issues, including transportation challenges, have been solved using ant colony techniques [20-23].

In this research, we examine Geometric Mean Combined and numerous adaptations of the ant colony algorithm utilizing Ant Colony Algorithm to Solve Multi-Objective Transportation Problems

in Fuzzy Environments to identify the optimal solution.

Preliminaries

In this section, some basic definitions of fuzzy numbers are presented. The Fuzzy set theory was first formulated by [24]. The following definitions of the fuzzy numbers and some operations on it may be useful [25].

Definition

A fuzzy set is a pair (X, μ_A) X is a set and $\mu_A: X \rightarrow [0,1]$. For all $x \in X$, $\mu_A(x)$ is called the membership function of x. If $\mu_A(x)=1$, we say that x is **Fully Included** in (X, μ_A) , and if $\mu_A(x)=0$, we say that x is **Not Included** in (X, μ_A) . If there exists some $x \in X$ such that $\mu_A(x)=1$, we say that (X, μ_A) is **Normal**. Otherwise, we say that (X, μ_A) is **Subnormal**.

Definition Triangular fuzzy number

A triangular fuzzy number (TFN) $\tilde{A}=(a_1, a_2, a_3)$ is FN with membership function

$$\mu_A(x) = \begin{cases} 0, & \text{for } x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\ 1, & \text{for } a_2 \leq x \leq a_3 \\ \frac{a_3-x}{a_3-a_1}, & \text{for } a_3 \leq x \leq a_3 \\ 0, & \text{for } x \geq a_3 \end{cases}$$

The α -cut of the TFN given by,

$A_\alpha = [a_1 + (a_2 - a_1)\alpha, a_3 + (a_2 - a_3)\alpha] \in (0, 1]$. Where a_1, a_2 and a_3 be real numbers with $a_1 \leq a_2 \leq a_3$.

Definition Geometric mean

The geometric mean is a mean or average that reveals the center tendency or typical value of a set of numbers by multiplying their values together (as opposed to the arithmetic mean which uses their sum) [26, 27]. In general, the geometric mean is defined as the n^{th} root of the product of n numbers, i.e., for a given set of numbers, the geometric mean is the nth root of the product of n numbers x_1, x_2, \dots, x_n , the geometric mean is defined as

$$(G x_i)^n = \sqrt[n]{x_1 x_2 x_3 \dots x_n}$$

Mathematical Model of Modified Ant Colony Algorithm (1991)

Here the new path based on the probability from place i to place j for the k-th ant as Shown in Equation (1)

$$P_{ij} = \frac{\Omega_{ij}^\theta \cup_{ij}^\varphi}{\sum_k \Omega_{ik}^\theta \cup_k^\varphi} \quad (1)$$

Where, Ω_{ij} and \cup_{ij} are the values of pheromone trail level of the move, and some heuristic information, which correspond to the link (i,j). θ And φ are both parameters used to control the importance of the pheromone trail and heuristic information during component selection. For our scenario we assumed $\Omega=1$ and $\varphi=1/3$ in the transition rule,

$$P_{ij}(t) = \begin{cases} \frac{(\prod_{j=1}^3 a_{ij})^{\frac{1}{3}}}{\sum_{i=1}^3 (\prod_{j=1}^3 a_{ij})^{\frac{1}{3}}} & ; \text{ } i^{\text{th}} \text{ ant visits the } j^{\text{th}} \text{ city} \\ 0 & ; \text{ Otherwise} \end{cases} \quad (2)$$

With

a_{ij} ; is cost between node i and node j.

$P_{ij}(t)$; Probability to branch from node i to node j

Pheromone Update Rule.

After all ants complete their tours, the local update rule of the pheromone trails is applied for each route according to (3),

$$\Omega_{ij}(t+1) = (1 - \rho)\Omega_{ij}(t) + \sum_{k=1}^m \Delta_{fk} \quad (3)$$

After that, apply the global pheromone update rule in which the amount of pheromone is added to the best route which has the lowest cost.

Here, Y^k is the distance of the best route. μ Is simply a parameter to adjust the amount of pheromone deposited, typically it would be set to 1. We sum μ/Y^k for every solution which used component (i, j), then that value becomes the amount of pheromone to be deposited on component (i, j). In our case ,

$$\Omega_{ij}(t+1) = (1 - \rho)\Omega_{ij}(t)$$

Where $\Omega_{ij}(t)$ is the maximum number of Demands or Supplies and $0 < \rho \leq 1$.

Mathematical Formulation

The mathematical formulation of the FTP is as follows.

$$\text{Minimize } \hat{c}_{ij} \hat{x}_{ij}$$

$$\begin{aligned} \text{Subject to} \\ \sum_{j=1}^n \hat{x}_{ij} &\leq a_i \text{ For } i = 1, 2, 3, \dots, m \\ \sum_{i=1}^m \hat{x}_{ij} &\leq b_j \text{ For } j = 1, 2, 3, \dots, n \\ \hat{x}_{ij} &\geq 0 \text{ For } i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n \end{aligned}$$

Here, all a_i and b_j are assumed to be positive, and a_i are normally called supplies and b_j are called demands, as shown in below table. The fuzzy cost \hat{c}_{ij} are all non-negative. If $\sum_{j=1}^n a_j = \sum_{i=1}^m b_i$, it is a balanced transportation problem .If this condition is not met, a dummy origin or destination is generally introduced to make the problem balanced.

Proposed Method

In this section, a proposed method, Improved Ant Colony Algorithm, for finding an optimal solution. Following are the steps for solving Fuzzy Transportation Problem.

Algorithm

Step 1: Construct the fuzzy transportation the cost table from the given problem

Step 2: Examine the TP to see if it is balanced, and if not, make it so.

Step 3: Convert fuzzy cost values in the Transportation cost table to crisp cost values by utilizing Geometric Mean.

Step 4: The probability table is then computed using the Modified ACO algorithm..

Step 5: Starting with the $X = \min(a_i, b_j)$ (unbalanced) or starting with the $X = \max(a_i, b_j)$ (balanced) probability table to make the first allocation.

Step 6: Assign, Step 5 at the place of the minimum probability cell

Step 7: If the demand in the column (or supply in the row) is satisfied, delete that column (or row), then proceed to the next minimal value in the demand and supply.

Step 8: Repeat this process until all supply and demand are satisfied, then proceed to Step – 10.

Otherwise, proceed to Step 6.

Step 9: Stop and compute the first viable solution.

Illustration Example

Example 1

We take a distribution problem in which a single homogeneous item is to be distributed from three stores (A, B, C) to four different warehouses (P, Q, and R, S). Cost, time and distance for each unit transported is given in the table. Find the minimum time, cost and distance.

Table 1: Cost

| Supply Demand | P | Q | R | S | Supply |
|---------------|----|----|----|----|--------|
| A | 21 | 16 | 15 | 13 | 11 |
| B | 17 | 18 | 24 | 23 | 13 |
| C | 32 | 27 | 18 | 41 | 19 |
| Demand | 6 | 10 | 12 | 15 | 43 |

Table 2: Time

| Supply Demand | P | Q | R | S | Supply |
|---------------|---|----|----|----|--------|
| A | 1 | 2 | 1 | 4 | 11 |
| B | 3 | 3 | 2 | 1 | 13 |
| C | 4 | 2 | 5 | 9 | 19 |
| Demand | 6 | 10 | 12 | 15 | 43 |

Table 3 :Distance

| Supply Demand | P | Q | R | S | Supply |
|---------------|----|----|----|----|--------|
| A | 11 | 13 | 17 | 14 | 11 |
| B | 16 | 18 | 14 | 10 | 13 |
| C | 21 | 24 | 13 | 10 | 19 |
| Demand | 6 | 10 | 12 | 15 | 43 |

Table 4: Step 3

| Supply Demand | P | Q | R | S | Supply |
|---------------|-------|-------|-------|-------|--------|
| A | 6.13 | 7.46 | 6.34 | 8.99 | 11 |
| B | 9.34 | 9.90 | 8.75 | 6.13 | 13 |
| C | 13.90 | 10.90 | 10.53 | 15.45 | 19 |
| Demand | 6 | 10 | 12 | 15 | 43 |

Table 5: Step 4

| Supply Demand | P | Q | R | S | Supply |
|---------------|------|------|------|------|--------|
| A | .053 | .065 | .055 | .078 | 11 |
| B | .082 | .086 | .076 | .053 | 13 |
| C | .122 | .095 | .092 | .135 | 19 |
| Demand | 6 | 10 | 12 | 15 | 43 |

Table 6: Steps 5 & 6 and 1st allocation

| Supply Demand | P | Q | R | S | Supply |
|---------------|------|------|---------|------|--------|
| A | .053 | .065 | .055 | .078 | 11 |
| B | .082 | .086 | .076 | .053 | 13 |
| C | .122 | .095 | .092*12 | .135 | 19*7 |
| | 6 | 10 | 12*0 | 15 | 43 |

Table 7: Steps 6 & 7 and 2nd allocation

| Supply Demand | P | Q | R | S | Supply |
|---------------|------|--------|---------|------|--------|
| A | .053 | .065 | .055 | .078 | 11 |
| B | .082 | .086 | .076 | .053 | 13 |
| C | .122 | .095*7 | .092*12 | .135 | 19*7*0 |
| | 6 | 10*3 | 12*0 | 15 | 43 |

Table 8: Steps 6 & 7 and 3rd allocation

| Supply Demand | P | Q | R | S | Supply |
|---------------|------|--------|---------|------|--------|
| A | .053 | .065*3 | .055 | .078 | 11*8 |
| B | .082 | .086 | .076 | .053 | 13 |
| C | .122 | .095*7 | .092*12 | .135 | 19*7*0 |
| | 6 | 10*3*0 | 12*0 | 15 | 43 |

Table 9: Steps 6, 7, 8 and 3rd allocation

| Supply Demand | P | Q | R | S | Supply |
|---------------|--------|--------|---------|------|--------|
| A | .053*6 | .065*3 | .055 | .078 | 11*8*2 |
| B | .082 | .086 | .076 | .053 | 13 |
| C | .122 | .095*7 | .092*12 | .135 | 19*7*0 |
| | 6*0 | 10*3*0 | 12*0 | 15 | 43 |

Table 9: Steps 6, 7, 8 and 3rd allocation

| Supply Demand | P | Q | R | S | Supply |
|---------------|--------|--------|---------|------|--------|
| A | .053*6 | .065*3 | .055 | .078 | 11*8*2 |
| B | .082 | .086 | .076 | .053 | 13 |
| C | .122 | .095*7 | .092*12 | .135 | 19*7*0 |
| | 6*0 | 10*3*0 | 12*0 | 15 | 43 |

Table 10: Steps 6, 7, 8 and 3rd allocation

| Supply Demand | P | Q | R | S | Supply |
|---------------|--------|--------|---------|--------|----------|
| A | .053*6 | .065*3 | .055 | .078*2 | 11*8*2*0 |
| B | .082 | .086 | .076 | .053 | 13 |
| C | .122 | .095*7 | .092*12 | .135 | 19*7*0 |
| | 6*0 | 10*3*0 | 12*0 | 15*13 | 43 |

Table 11: Step 9

| Supply Demand | P | Q | R | S | Supply |
|---------------|--------|--------|---------|---------|----------|
| A | .053*6 | .065*3 | .055 | .078*2 | 11*8*2*0 |
| B | .082 | .086 | .076 | .053*13 | 13*0 |
| C | .122 | .095*7 | .092*12 | .135 | 19*7*0 |
| | 6*0 | 10*3*0 | 12*0 | 15*13*0 | 43 |

The solution is given as: $x_1 = 6, x_2 = 3, x_3 = 2, x_4 = 13, x_5 = 7, x_6 = 12$

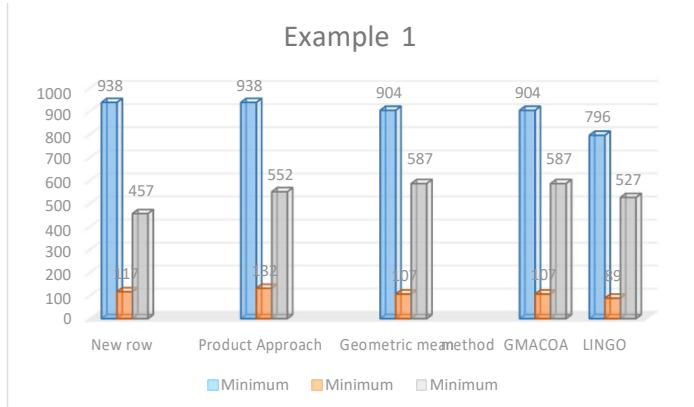
$11 \quad 12 \quad 13 \quad 24 \quad 32 \quad 33$

Following are the values of objectives: Minimum Cost = 904 units, Minimum Time = 107 units, Minimum distance = 587 units (1 iteration)

Table 12: Comparison between different methods

| Method | Minimum cost | Minimum time | Minimum Distance |
|---------------------------|--------------|--------------|------------------|
| New row Maxima method [9] | 938 | 117 | 457 |
| Product Approach [10] | 938 | 132 | 552 |
| Geometric mean method | 904 | 107 | 587 |
| GMACOA | 904 | 107 | 587 |
| LINGO | 796 | 89 | 527 |

The findings of the comparisons in Table 12 are depicted using bar graphs, as shown in Fig. 1.


Figure 1: Compares the results of the New Row Maxima technique, the Product Approach, the Geometric Mean method, GMACOA, and the optimal method (LINGO).

According to the simulation findings (Fig. 1 and Table 12), the proposed strategy outperforms the geometric mean method.

Example 2(Khilendra,2020)

Now we take one more example with following characteristics:

Table 13: Cost

| Supply Demand | P | Q | R | S | Supply |
|---------------|---|----|----|---|--------|
| A | 6 | 4 | 1 | 5 | 14 |
| B | 8 | 9 | 2 | 7 | 16 |
| C | 4 | 3 | 6 | 2 | 5 |
| Demand | 6 | 10 | 15 | 4 | |

Table 14: Time

| Supply Demand | P | Q | R | S | Supply |
|---------------|----|----|----|----|--------|
| A | 13 | 11 | 15 | 20 | 14 |
| B | 17 | 14 | 12 | 13 | 16 |
| C | 18 | 18 | 15 | 12 | 5 |
| Demand | 6 | 10 | 15 | 4 | |

Table 15: Distance

| Supply Demand | P | Q | R | S | Supply |
|---------------|---|----|----|---|--------|
| A | 6 | 3 | 5 | 4 | 14 |
| B | 5 | 9 | 2 | 7 | 16 |
| C | 5 | 7 | 8 | 6 | 5 |
| Demand | 6 | 10 | 15 | 4 | |

Table 16: Step 3

| Supply Demand | P | Q | R | S | Supply |
|---------------|------|-------|------|------|--------|
| A | 7.76 | 5.09 | 4.21 | 7.36 | 14 |
| B | 8.79 | 10.42 | 3.63 | 8.60 | 16 |
| C | 7.11 | 7.23 | 8.96 | 5.24 | 5 |
| Demand | 6 | 10 | 15 | 4 | |

Table 17: Step 4

| Supply Demand | P | Q | R | S | Supply |
|---------------|------|------|------|------|--------|
| A | .091 | .060 | .049 | .087 | 14 |
| B | .104 | .123 | .043 | .101 | 16 |
| C | .084 | .085 | .106 | .062 | 5 |
| Demand | 6 | 10 | 15 | 4 | |

Table 18: Step 5 and 1st allocation

| Supply Demand | P | Q | R | S | Supply |
|---------------|------|------|---------|------|--------|
| A | .091 | .060 | .049 | .087 | 14 |
| B | .104 | .123 | .043*15 | .101 | 16*1 |
| C | .084 | .085 | .106 | .062 | 5 |
| Demand | 6 | 10 | 15*0 | 4 | |

Table 19: Steps 5, 6, 7 and 1st allocation

| Supply Demand | P | Q | R | S | Supply |
|---------------|------|---------|---------|------|--------|
| A | .091 | .060*10 | .049 | .087 | 14*4 |
| B | .104 | .123 | .043*15 | .101 | 16*1 |
| C | .084 | .085 | .106 | .062 | 5 |
| Demand | 6 | 10*0 | 15*0 | 4 | |

Table 20: Steps 5-8 and other allocations

| Supply Demand | P | Q | R | S | Supply |
|---------------|---------|---------|---------|--------|--------|
| A | .091*4 | .060*10 | .049 | .087 | 14*4*0 |
| B | .104*1 | .123 | .043*15 | .101 | 16*1 |
| C | .084*1 | .085 | .106 | .062*4 | 5*1 |
| Demand | 6*5*1*0 | 10*0 | 15*0 | 4*0 | |

Table 21: Comparison between different methods

| Method | Minimum cost | Minimum time | Minimum Distance |
|-----------------------|--------------|--------------|------------------|
| New row | 122 | 461 | 130 |
| Maxima method [9] | | | |
| Product Approach [10] | 114 | 425 | 128 |
| Geometric mean method | 114 | 425 | 118 |
| GMACOA | 114 | 425 | 118 |
| LINGO | 114 | 424 | 106 |

Table 21's comparison data are also depicted using bar graphs and the results Fig. 2.

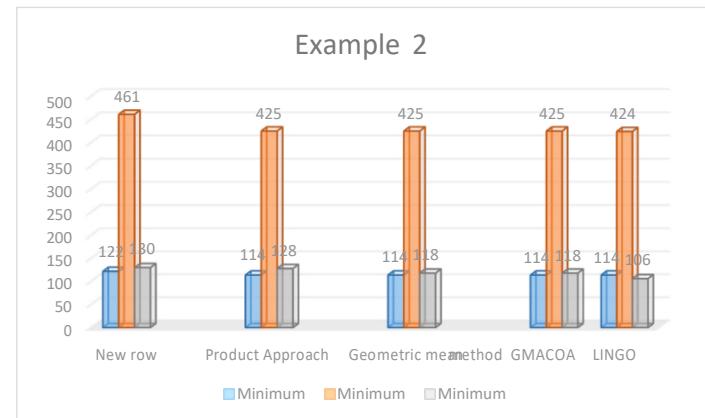


Figure 2: Shows a comparison of the outcomes given by the New Row Maxima approach, the Product Approach, the Geometric Mean method, GMACOA, and the optimal method.

The proposed technique uses the geometric mean method, according to the simulation results (Fig. 2 and Table 21).

Example 3
Table 22: Example

| x | D ₁ | D ₂ | D ₃ | Supply |
|----------------|----------------|----------------|----------------|--------|
| S ₁ | (1,2,3) | (4,7,10) | (10,14,18) | 5 |
| S ₂ | (2,3,4) | (2,3,4) | (0,1,2) | 8 |
| S ₃ | (1,5,9) | (3,4,5) | (4,7,10) | 7 |
| S ₄ | (0,1,2) | (5,6,7) | (1,2,3) | 15 |
| Demand | 7 | 9 | 18 | |

Table 22: Step 2

| x | D ₁ | D ₂ | D ₃ | Dummy | Supply |
|----------------|----------------|----------------|----------------|-------|--------|
| S ₁ | (1,2,3) | (4,7,10) | (10,14,18) | 0 | 5 |
| S ₂ | (2,3,4) | (2,3,4) | (0,1,2) | 0 | 8 |
| S ₃ | (1,5,9) | (3,4,5) | (4,7,10) | 0 | 7 |
| S ₄ | (0,1,2) | (5,6,7) | (1,2,3) | 0 | 15 |
| Demand | 7 | 9 | 18 | 1 | |

Table 24: Step 6 and 1st allocation

| x | D ₁ | D ₂ | D ₃ | Dummy | Supply |
|----------------|----------------|----------------|----------------|-------|---------|
| S ₁ | .036*4 | .132 | .275 | 0*1 | 5*4*0 |
| S ₂ | .058 | .058 | 0 | 0 | 8 |
| S ₃ | .071 | .079 | .132 | 0 | 7 |
| S ₄ | 0*3 | .120 | .036*12 | 0 | 15*12*0 |
| Demand | 7*3*0 | 9 | 18 | 1 | |

Table 23: Step 3

| x | D ₁ | D ₂ | D ₃ | Dummy | Supply |
|----------------|----------------|----------------|----------------|-------|--------|
| S ₁ | 1.81 | 6.54 | 13.61 | 0 | 5 |
| S ₂ | 2.88 | 2.88 | 0 | 0 | 8 |
| S ₃ | 3.55 | 3.91 | 6.54 | 0 | 7 |
| S ₄ | 0 | 5.94 | 1.81 | 0 | 15 |
| Demand | 7 | 9 | 18 | 1 | |

Table 25: Other steps and other allocations

| x | D ₁ | D ₂ | D ₃ | Dummy | Supply |
|----------------|----------------|----------------|----------------|-------|---------|
| S ₁ | .036*4 | .132 | .275 | 0*1 | 5*4*0 |
| S ₂ | .058 | .058*2 | 0*6 | 0 | 8*2*0 |
| S ₃ | .071 | .079*7 | .132 | 0 | 7*0 |
| S ₄ | 0*3 | .120 | .036*12 | 0 | 15*12*0 |
| Demand | 7*3*0 | 9*7*0 | 18 | 1 | |

The solution is given as: $x_{11}=4$, $x_{22}=2$, $x_{23}=6$, $x_{32}=7$, $x_{41}=3$, $x_{43}=12$
 Following are the values of objectives: $4(1,2,3)+2(2,3,4)+6(0,1,2)+7(3,4,5)+3(0,1,2)+12(1,2,3)=(41,75,109)=75$

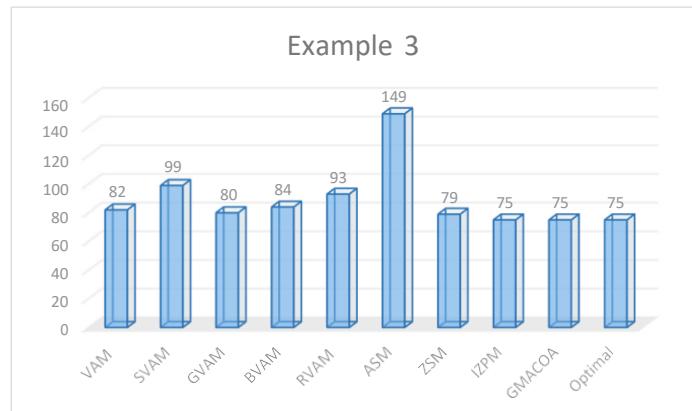
Table 23: Step 4

| x | D ₁ | D ₂ | D ₃ | Dummy | Supply |
|----------------|----------------|----------------|----------------|-------|--------|
| S ₁ | .036 | .132 | .275 | 0 | 5 |
| S ₂ | .058 | .058 | 0 | 0 | 8 |
| S ₃ | .071 | .079 | .132 | 0 | 7 |
| S ₄ | 0 | .120 | .036 | 0 | 15 |
| Demand | 7 | 9 | 18 | 1 | |

Table 24: Initial Solutions Obtained by all Procedures

| Method | VAM | SVAM | GVAM | BVAM | RVAM | ASM | ZSM | IZPM | GMA-COA | Optimal |
|-----------|-----|------|------|------|------|-----|-----|------|---------|---------|
| Example 3 | 82 | 99 | 80 | 84 | 93 | 149 | 79 | 75 | 75 | 75 |

The comparison data from Table 24 are also represented using bar graphs and the form Fig. 3.


Figure3: A comparison of the results obtained by VAM,SVAM,G-

VAM,BVAM,RVAM,ASM,ZSM,IZPM,GMACOA,AND Optimal According to the simulation results (Fig. 3 and Table 24), the suggested strategy employs the IZPM method, and other ways outperform our GMACOA. In addition, the shortest number of iterations resulted in the best solution.

Conclusion

This serious cosmos cannot exist without the TP. The primary goal of standard TPs is to lessen the expense of transporting a product from its point of origin to its final destination. Some significant challenges need the simultaneous examination and optimisation of many goals. These kind of issues are called multi-objective problems. This study tackles a multi-objective fuzzy transportation issue in fuzzy settings by combining the Ant Colony Algorithm with the geometric mean technique, rather than relying on traditional methodologies. The suggested algorithm offers the most performance when compared with other existing methods. Therefore,

The number of iterations needed to get the optimal solution decreases as the IFS increases [28, 33]. It is a really simple procedure. In contrast, this research presents a fresh other approach—a tweaked ant colony optimisation algorithm—that offers a perfect answer to the various TP kinds.

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