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Investigating the Oscillatory Behavior in Third-Order Delay Difference Equations with Non-Canonical Operators

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Abstract:

Oscillatory criteria for third-order delay difference equations with non-canonical operators are introduced in this study. One may find new adequate conditions for the oscillation of all equation solutions by using the Riccati transformation approach. Some of the findings presented in the literature have been improved upon and expanded upon by established outcomes. The significance of the key findings is shown by providing examples.

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Keywords and Phrases: Oscillatory Behavior, delay difference equation, third -order.

1 Introduction

This paper deals with the oscillatory behaviour of solutions to third order delay difference equations of the form

$$\Delta(a_n(\Delta(b_n\Delta x_n))^{\alpha}) + p_n x_{n-k}^{\alpha} = 0\,, n \geq n_0 > 0 \qquad (1.1)$$

(H1) $\{a_n\}$ & $\{b_n\}$ is a positive real sequence for all $n \geq n_0$ such that $A_{n_0} = \sum_{n=n_0}^{\infty} \frac{1}{b_n} < \infty$

& $B_{n_0} = \sum_{n=n_0}^{\infty} \frac{1}{a_n^{\frac{1}{n}}} < \infty$

(H2) $\{p_n\}$ are positive real sequences for all $n \geq n_0$ and $p_n \neq 0$ for infinite valuesof n;

(H3) α is a ratio of odd positive integers and k is a positive integer;

(H4) $\alpha \in (0,1], \beta$ and γ are ratio of odd positive integers.

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A solution of equation (1.1) , means a real sequence $\{x_n\}$ satisfying equation (1.1) for all $n \geq n_0 - k$. It is easy to see that under the initial conditions

$$x_n = \emptyset , n = n_0 - \mathrm{k} , n_0 - \mathrm{k} + 1, \ldots \ldots \ldots . n_0 \qquad (1.2)$$

are given, then equation(1.1) has a unique solution satisfying (1.2)

A nontrivial solution of equation (1.1) is said to be oscillatory if the terms of the sequence are neither positive nor eventually negative and non oscillatory otherwise.

The equation itself is called oscillatory if all its solutions oscillate. From the discrete kneser's theorem [1] ,the equation (1.1) has property A if any solution x_n of (1.1) is either oscillatory or tends to zero as $n \to \infty$.

The investigation of oscillatory and asymptotic properties of equation (1.1) is important for applications, since such equations arise in the study of economics, mathematical biology and many other areas of applied mathematics and physics, see [1, 6]. In the last three decades the oscillation theory of difference equations has been extensively developed, see for example [1, 2,5,7,8,9,11,14,15,16], and the references cited therein. From the review of literature, we can see that several oscillation criteria are provided under the conditions.

Delay differential equations are a type of differential equation in which the derivative of the unknown function at a certain time given in terms of the values of the function at previous times. DDE's are also called time –delay systems, system with after effect or dead-time. A general form of time-delay differential equations for $x(t) \in R^n$ is $\frac{d}{dt}(t) = (t, (t), x_t)$, where $x_t = \{x(r) : r \leq t\}$ represents the trajectory of the solution in the past. In this equation ,f is a functional operator from $R \times R^n \times C^1(R, R^n)$ to R^n.

A delay differential equation where the state variable appears with delayed argument.

Being aware of numerous indications of the practical importance of third-order differential equations as well as a number of mathematical problems involved in the area of the qualitative theory for such equations has attracted a large portion of research interest in the last three decades. The asymptotic properties of equations were extensively investigated in the literature, see, e.g., [3–14] and the references cited therein. Most of the papers have been devoted to the examination of so-called canonical equations, The advantage and usefulness of a non canonical representation of linear disconjugate operators in the study of the oscillatory

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and asymptotic behavior of (1) was recently shown in [15]. In 2018, Dzurina and Jadlovská [16] considered a particular case of Equation (1) in non canonical form with p ≡ 0 and established various oscillation criteria for Equation (1). Their method simplifies the process and reduces the number of conditions required in previously known results. A further improvement of these results was presented in [17]. Depending on various ranges of p, a variety of results for property A of (1), its generalizations or particular cases, exist in the literature, and the references cited.

2 Main Results

We provide new criterion for oscillation in equation (1.1) in this section. The standard operating procedure in this work is to assume that all functional inequalities are true for every sufficiently big n. We may limit ourselves to dealing with positive solutions of equation (1.1) without sacrificing generalizability. Possible non-oscillatory solutions of equation (1.1) are first considered in terms of their structure.

Theorem 2.1

Assume (H_1) - (H_3) hold. If

$$\lim_{n\to\infty}\inf\sum_{s=n-k}^{n-1}\frac{1}{b_s}\sum_{t=N}^{s-1}\left(\frac{1}{a_t}\sum_{j=N}^{t-1}p_j\right)^{\frac{1}{\alpha}} > \left(\frac{k}{k+1}\right)^{k+1} \tag{2.11}$$

and

$$\lim_{n\to\infty}\operatorname{Sup}\sum_{s=n-k}^{n-1}\frac{1}{b_s}\sum_{t=s}^{s-1}\left(\frac{1}{a_t}\sum_{j=t}^{n-1}p_j\right)^{\frac{1}{\alpha}} > 1 \tag{2.12}$$

then every solution of equation (1.1) is oscillatory.

Let $\{x_n\}$ be a non oscillatory solution of equation (1.1) for all , $n \geq n_0$. Without loss of generality, we may assume that $x_n > 0$ and $x_{n-k} > 0$ for all $n \geq N \geq n_0$Then there are four possible Cases (I)- (II)

Case (I)

Summing equation (1.1) from N to n-1 and using the fact the $\{x_n\}$ is decresing

$$-a_n\left(\Delta\left(b_n\Delta x_n\right)\right)^{\alpha} \geq x_{n-k}^{\alpha}\sum_{s=N}^{n-1}p_n \tag{2.13}$$

Or

$$-\Delta\left(b_n\Delta x_n\right) \geq x_{n-k}\left(\frac{1}{b_n}\sum_{s=N}^{n-1}p_n\right)^{\frac{1}{\alpha}} \tag{2.14}$$

is obtained. Summing again from N to n-1, the following is obtained

$$-b_n \Delta x_n \geq \sum_{s=N}^{n-1} x_{s-k} \left(\frac{1}{b_s} \sum_{t=N}^{s-1} p_t\right)^{\frac{1}{\alpha}} \quad (2.15)$$

$$\text{Or } \Delta x_n + \left(\frac{1}{b_n} \sum_{s=N}^{n-1} \left(\frac{1}{a_s} \sum_{s=N}^{s-1}\right) x_t^{\frac{1}{}} {}_{n-k}\right) \leq 0. \quad (2.16)$$

However by known result (2.11) implies that the above inequality does not possess a positive solution, which is a contradiction.

Case (II)

Summing equation (1.1) from $o\ (n-1)(>j)$ and using the fact that $\{x_n\}$ isdecreasing ,

$$a_n\left(\Delta\left(b_n \Delta x_n\right)\right)^{\alpha} \geq x_{n-k}^{\alpha} \sum_{s=j}^{n-1} q_s \quad (2.17)$$

$$\text{Or } \Delta(a_n \Delta x_n) \geq x_{n-k} \left(\frac{1}{a_n} \sum_{s=j}^{n-1} p_s\right)^{\frac{1}{\alpha}} \quad (2.18)$$

is arrived. If the above process of summation from $j\ to\ (n-1)(>j)$ is repeated two times , the following is obtained

$$x_j \geq x_{n-k} \sum_{s=j}^{n-1} \frac{1}{b_s} \sum_{t=s}^{n-1} \left(\frac{1}{a_t} \sum_{i=t}^{n-1} p_i\right)^{\frac{1}{\alpha}} \quad (2.19)$$

Letting $j = n-k$ in (2.19) ,contradiction with (2.12) is attained.

Hence, $\{x_n\}$ be a non oscillatory solution of equation (1.1) for all , $n \geq n_0$ is wrong.

That is every solution of equation (1.1) is oscillatory.

3 Example

Consider the third order delay difference equation

$$\Delta(2^n \Delta(2^n \Delta x_n)) + 30\ \ 4^n\ x_{n-3} = 0\ , n \geq 1 \quad (3.1)$$

After simple computations, conditions (2.11) and (2.12) are satisfied.

Therefore by the above theorem, every solution of equation (3.1) is Oscillatory.

It is important to note that none of the results reported in the literature can yield this conclusion.

4 Conclusion

Solutions of the half-linear third-order delay difference equation (1.1) with non-canonical operators are examined in this work for their oscillatory features. To start, all solutions to equation (1.1) are shown to oscillate using just two condition criteria, as opposed to the three or more requirements often utilised in the literature. Oscillatory characteristics of non-canonical third order delay difference equations may be investigated with relative ease using the suggested novel approach, and the requirements can be easily verified.

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