



IJITCE

ISSN 2347- 3657

International Journal of Information Technology & Computer Engineering

www.ijitce.com



Email : ijitce.editor@gmail.com or editor@ijitce.com

Investigating the Vertex Fixed Odd Geo-Domination Number in Graph Theory

Mrs. **Sudha Madhavi Bitta¹**, **Velkanti Shravani²**

*1 Assistant Professor, Department of H&S, Malla Reddy College of Engineering for Women.,
Maisammaguda., Medchal., TS, India
2, B.Tech ECE (20RG1A0453),
Malla Reddy College of Engineering for Women., Maisammaguda., Medchal., TS, India*

Abstract-Let x be any vertex of a connected graph G of order $n \geq 3$. A set $S \subseteq V$ of a graph G is said to be an x - odd geo-dominating set if for every vertex $v \in V - (S \cup \{x\})$ must lies in $x - y$ geodesic for some $y \in S$ and $|N(v) \cap (S \cup \{x\})| \equiv 1(mod 2)$. The minimum cardinality of the x - odd geo-dominating set is called x - odd geo-domination number denoted by $g_{x-odd}(G)$. The x - odd geo-dominating set with cardinality $g_{x-odd}(G)$ is called g_{x-odd} - set of G . In this paper we determine the bounds for a vertex fixed odd geodomination number and the same for some families of graphs It is shown that every pair k, n of integers with $1 \leq k \leq n - 1, n \geq 3$,

there exists a connected graph G of order n and $g_{x-odd}(G) = k$

Key words: geodesic, geodominating set, odd geo-dominating set, odd geo-domination number.

I. INTRODUCTION

Geodetic number ideas have several intriguing uses in solving the shuttle and network route design issue [2, 5]. Other fields that make use of geodetic number principles include data mining, neural networks, picture and video editing, distributed computing, facility placement, and telephone switching centres. The terms "geodetic sets" and "geodomination number" are borrowed from [4] and used consistently throughout the article. A novel definition of a graph's vertex fixed odd geo-domination number was inspired by the work of [1].

II. MAIN RESULT

Definition 2.1 Let x be any vertex of a connected graph G of order $n \geq 3$. A set $S \subseteq V$ of a graph G is said to be an x - odd geo-dominating set if for every vertex $v \in V - (S \cup \{x\})$ must lies in $x - y$ geodesic for some $y \in S$ and $|N(v) \cap (S \cup \{x\})| \equiv 1(mod 2)$. The minimum cardinality of the x - odd geo-dominating set is called x - odd geo-domination number denoted by $g_{x-odd}(G)$. The x - odd geo-dominating set with cardinality $g_{x-odd}(G)$ is called g_{x-odd} - set of G .

Example 2.2 For the graph G given in Figure 1, the minimum x - odd geo-dominating set and x - odd geo-domination number $g_{x-odd}(G)$ are given in the Table 1.

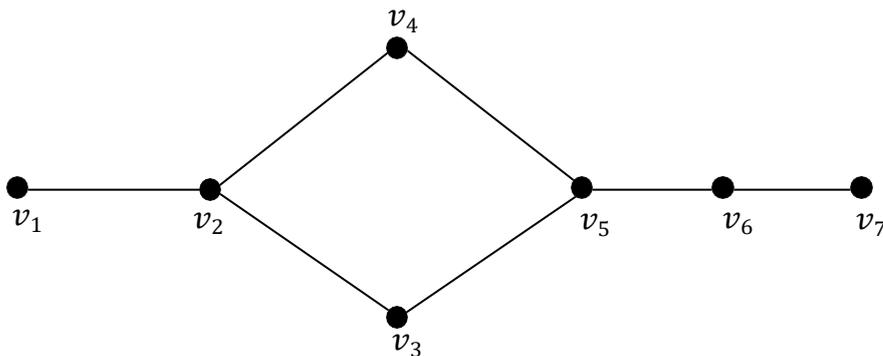


Figure 1: A Graph G

Vertex x	x - odd geo-dominating set	$g_{x-odd}(G)$
v_1	$\{v_5, v_6, v_7\}$	3
v_2	$\{v_1, v_6, v_7\}$	3
v_3	$\{v_1, v_4, v_6, v_7\}$	4
v_4	$\{v_1, v_3, v_6, v_7\}$	4
v_5	$\{v_1, v_6, v_7\}$	3
v_6	$\{v_1, v_2, v_6\}$	3
v_7	$\{v_1, v_5, v_6\}$	3

Table 1

Theorem 2.3 For any vertex in a connected graph G of order $n \geq 3$, the vertex x does not belongs to any minimum x - odd geo-dominating set of G .

Proof: Suppose that x belongs to minimum x - odd geo-dominating set S of G . Let G be a graph of order $n \geq 3$. Since $n \geq 3$ and by definition 2.1, there exists a vertex $v \neq x$ belongs to S . Clearly x lies in every $x - v$ geodesic. Therefore $T = S - \{x\}$ is an x - odd geo-dominating set of G , which is a contradiction to S is a minimum x - odd geo-dominating set of G .

Theorem 2.4 Every extreme vertex must belong to every g_{x-odd} - set. In particular, every end vertex must belong to every g_{x-odd} - set.

Proof: By theorem 2.3, the vertex x does not belongs to any g_{x-odd} - set, say S . Let $u \neq x$ be an extreme vertex of G . Suppose $u \notin S$. Then clearly u must lie in some $x - v$ geodesic, $v \in S$ and $|N(u) \cap (S \cup \{x\})| \equiv 1 \pmod{2}$. That is, u must be adjacent to odd number of vertices, say w_1, w_2, \dots, w_k , where $k = \text{odd}$ of $S \cup \{x\}$. Suppose u is adjacent to $w_i \neq x$. Then clearly there exists a $x - v$ geodesic, say $x, \dots, w_i, u, w_j, \dots, v$. It implies that u is adjacent to non - adjacent vertices, which is a contradiction to u is an extreme vertex. Suppose u is adjacent to w_i 's and $w_i = x$. Then clearly there exists a $x - v$ geodesic, say x, u, w_i, \dots, v . It implies that u is adjacent to non - adjacent vertices, which is a contradiction to u is an extreme vertex. Hence the result.

Corollary 2.5 Let K_n be a complete graph. Then $g_{x-odd}(K_n) = n - 1$.

Proof: Let x be any vertex of $V(K_n)$. Since all the vertices are extreme vertex in K_n , by theorem 2.3 and 2.4, $g_{x-odd}(K_n) = n - 1$.

Proposition 2.6 For any vertex x in a connected graph G of order $n \geq 2$, $1 \leq g_{x-odd}(G) \leq n - 1$.

Proof: By the definition of g_{x-odd} - set, $g_{x-odd}(G) \geq 1$. Also by theorem 2.3, the vertex x does not belongs to any g_{x-odd} - set. Thus $g_{x-odd}(G) \leq n - 1$.

Remark 2.7 The bounds in the above proposition are sharp. If G is a complete bipartite $K_{2,n}$, $n \geq 2$ with partition U and W , then $g_{x-odd}(G) = 1$ for any x in U . For a complete graph K_n ($n \geq 2$), $g_{x-odd}(K_n) = n - 1$.

Theorem 2.8 Let G be a complete bipartite graph $K_{m,n}$ of order $p = m + n \geq 4$ ($m, n \geq 2$). Let V_1 and V_2 be a partition of $V(G)$. Then $g_{x-odd}(G) = |V_i| - 1$ if $x \in V_i$ and $|V_i| = \text{odd}$ and $g_{x-odd}(G) = p - 1$ if $x \in V_i$ and $|V_i| = \text{even}$, $i = 1, 2$.

Proof: Let $V_1 = \{v_1, v_2, \dots, v_m\}$ and $V_2 = \{u_1, u_2, \dots, u_n\}$ be the partition of $V(G)$.

Case (i) Let $x \in V_i$ and $|V_i| = \text{even}$, $i = 1, 2$.

Take $S = \{u_i\}$, $u_i \neq x \in V_i$. Then clearly all the vertices in another partition V_j , $i \neq j$ lies in $x - u_i$ geodesic. But the vertices in V_i other than x does not lies in any geodesic. So S is not a x - odd geo-dominating set. Take $S' = S \cup \{v_i, v_j\}$, $v_i, v_j \in V_j$. Clearly it does not forms a x - odd geo-dominating set, since all vertices in V_i is adjacent to v_i and v_j and so no vertices of V_i does not lies in $x - u_i$ geodesic. Take $S'' = \{\text{set of all vertices in } V_i \text{ other than } x\}$. Clearly all the vertices in $V - (S'' \cup \{x\})$ lies in $x - u_i$ geodesic for some $u_i \in S''$ and $|N(v_i) \cap (S'' \cup \{x\})| \equiv 1 \pmod{2}$, since $|V_i|$ is even. So clearly the whole vertex set other than x forms a x - odd geo-dominating set. Also it is minimum. Hence $g_{x-odd}(G) = p - 1$, $i = 1, 2$.

Case (ii) Let $x \in V_i$ and $|V_i| = \text{odd}$, $i = 1, 2$.

Take $S = \{v_i\}$, $v_i \neq x \in V_i$. Then clearly all the vertices in another partition V_j , $i \neq j$ lies in $x - v_i$ geodesic, since all the vertices in the partition V_1 is adjacent to V_2 and vice versa. Also $|(u_i) \cap (S \cup \{x\})| \equiv 1 \pmod{2}$. But the vertices in V_i other than x does not lies in any geodesic. Take $S' = \cup \{u_i, u_j\}$, $i, u_j \in V_j$. Clearly it does not forms a x - odd geo-dominating set, since all vertices in V_i is adjacent to u_i and u_j and so no vertices of V_i does not lies in $x - u_i$ geodesic. Take $S'' = \{\text{set of all vertices in } V_i \text{ other than } x\}$. Clearly all the vertices in $V - (S'' \cup \{x\})$ lies in $x - v_i$ geodesic for some $v_i \in S''$ and $|N(u_i) \cap (S'' \cup \{x\})| = |V_j| \equiv 1 \pmod{2}$, since $|V_j|$ is odd and all vertices in V_i is adjacent to V_j and vice versa. Also it is minimum. Hence $g_{x\text{-odd}}(G) = |V_i| - 1$, $i = 1, 2$.

Theorem 2.9 For any connected graph G , $1 \leq g_{x\text{-odd}}(G) < g_{\text{odd}}(G) \leq n$.

Proof: By the definition of $g_{x\text{-odd}}$ - set, we need at least one vertex for a x - odd geo-dominating set. Hence $g_{x\text{-odd}}(G) \geq 1$. Also by definition of odd geo-dominating set and by theorem 2.3, $g_{x\text{-odd}}(G) < g_{\text{odd}}(G)$. Since all the vertices of G forms an odd geo-domination set of G , $g_{\text{odd}}(G) \leq n$. Hence $1 \leq g_{x\text{-odd}}(G) < g_{\text{odd}}(G) \leq n$.

Theorem 2.10 For every pair k, n of integers with $1 \leq k \leq n - 1, n \geq 3$, there exists a connected graph G of order n and $g_{x\text{-odd}}(G) = k$.

Proof: Case (i) Assume that $k = n - 1$. Take $G = K_n$. Then by corollary 2.5, $g_{x\text{-odd}}(G) = n - 1$.

Case (ii) Assume that $1 \leq k < n - 2$. Let us consider a path $P: a, b, c, x$. Let G be a new graph obtained by joining $k - 2$ new vertices v_1, v_2, \dots, v_{k-1} to x and by joining $n - k - 3$ new vertices $u_1, u_2, \dots, u_{n-k-3}$ to both b and x as shown in the Figure 2.

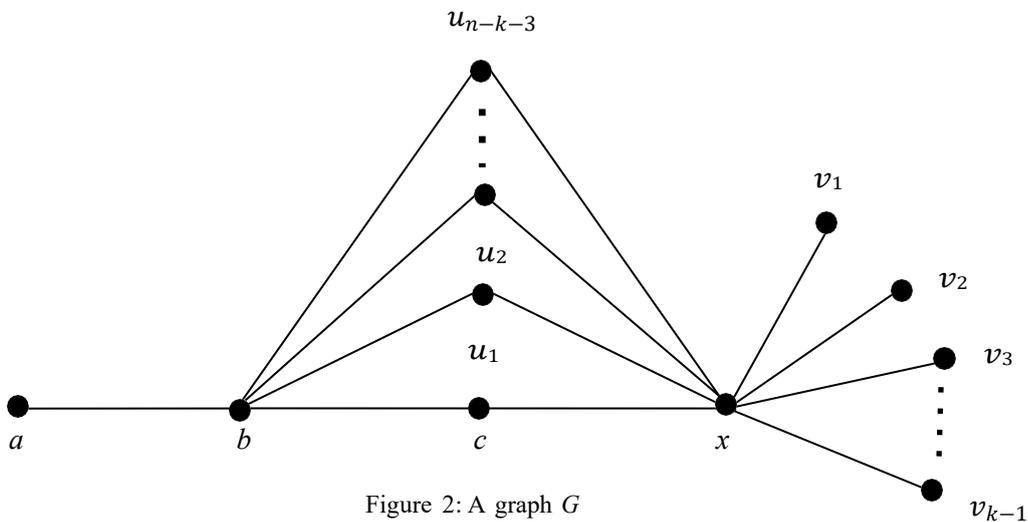


Figure 2: A graph G

Take $S = \{v_1, v_2, \dots, v_{k-1}\}$. Since by theorem 2.4, the vertices in S belong to a $g_{x\text{-odd}}$ - set. But clearly, S is not an x - odd geo-dominating set. Take $S' = S \cup \{a\}$. For every $v \in V - S'$, $|(v) \cap (S' \cup \{x\})| = 1 \equiv 1 \pmod{2}$ and so it is a x -odd geo-dominating set and it is minimum. Thus $g_{x\text{-odd}}(G) = |S'| = k - 1 + 1 = k$ and order of G is $4 + (n - k - 3) + (k - 1) = n$.

Case (iii) Assume that $k = n - 2$, $n \geq 4$

Sub case (a) If $n = \text{even}$, then we consider a path $P_2: v, x$. Let G be a new graph obtained by joining k new vertices v_1, v_2, \dots, v_k to v . The new graph G hence obtained is shown in Figure 3. By theorem 2.4, all external vertices other than x belong to a minimum x -odd geo-dominating set, say S . Clearly, v lies in the $x - v_i$ geodesic. Also, $|(v) \cap (S \cup \{x\})| \equiv 1 \pmod{2}$, since k is even and these k vertices along with x , $N(v)$ contains $k + 1$ vertices, which is odd. S is an odd geo-dominating set of G and it is minimum. Thus $g_{x\text{-odd}}(G) = |S| = k$ and order of G is $2 + k = (2 + n - 2) = n$.

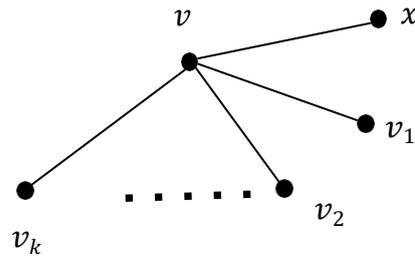


Figure 3: A Graph G

Sub case (b) If $n = \text{odd}$, then consider a path $P: y, x, z$ of order 3. Add $k - 2$ new vertices v_1, v_2, \dots, v_{k-1} to P and join each v_i ($1 \leq i \leq k-1$) to the vertex z . The new graph G hence obtained is shown in Figure 4.

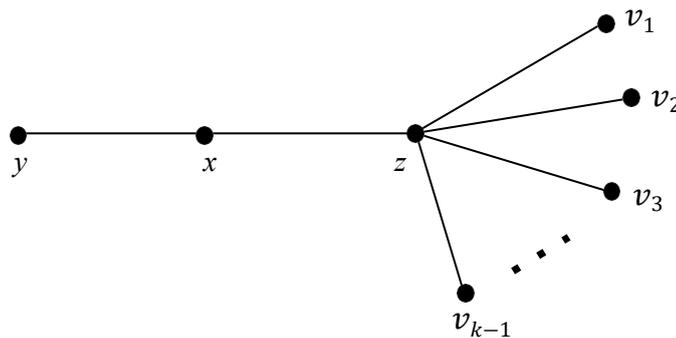


Figure 4: A Graph G

In this case, we claim that $g_{x-}(G) = k$. Let $S = \{y, v_1, v_2, \dots, v_{k-1}\}$ be the set of all extreme vertices of G . By theorem 2.4, the vertices of S must be in g_{x-odd} -set. Since $|(z) \cap (S \cup \{x\})| = k - 2 \equiv 1 \pmod{2}$, S is an x -odd geo-dominating set of G and so $g_{x-odd}(G) = |S| = k - 1 + 1 = k$ and the order of G is $(k - 1) + 3 = k + 2 = n$.

III. CONCLUSION

An odd geo-domination number is defined for vertices in this work. Bounds and realisation findings have been our primary topics of discussion. Additionally, we are now investigating algorithms that can calculate the set of geo-dominating graphs with fixed odd vertices. Furthermore, we are investigating the topic of vertex odd geo-domination numbers in networks that are connected to coronas.

REFERENCES

- [1] A. Anto Kinsley and K. Karthika, 2020, "Odd Geo-dominating Number of a Graph", Science, Technology and Development Journal, Volume IX, Issue XII, 442-447, ISSN: 0950-0707.
- [2] F. Buckley and F. Harary, *Distance in Graphs*, Addition – Wesley, Redwood City, CA, 1990.
- [3] G. Chartrand, F. Harary and P. Zang, *On the geodetic number of a graph*, Networks. 39(2002) 1-6.
- [4] G. Chartrand, F. Harary, H. Sart, and P. Zang, *Geodomination in graphs*, Bull. Inst. Appl. 31(2001), 51-59.
- [5] F. Harary, *Graph Theory*, Addition – Wesley, 1969.
- [6] F. Harary, E. Loukakis, C. Tsourous, *The geodetic number of a graph*, Math. Comput. Modeling, 17(11)(1993), 87-95.