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Complete lift of Affine Connection to the Complex Tangent Bundle

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Abstract

In this paper, complete lift of affine connection on the complex manifold M to its tangent bundle TM are studied.

Introduction:

Several authors have introduced complete lift on the tangent bundle TM of a smooth manifold M using notations of complete lifts on manifold M , but no natural conjecture has been presented for study of complex

structure on tangent bundle. This demands introduction of some new construction, which we shall prefer to call the construction of complex analytic tangent bundle of a complex manifold and in brief complex tangent bundle by TM_{2n} .

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2. Complete Lifts of Affine connection on TM_{2n} :

Let ∇ be the covariant differentiation of affine connection on complex manifold M_{2n} , and then there exist a unique affine connection of TM_{2n} whose covariant differentiation ∇^C satisfies

$$\nabla_{z^c}^C (U^C) = (\nabla_z U)^C$$

where $Z, U \in \mathfrak{X}_0(M_{2n})$.

Let $\Gamma_{\beta\gamma}^\alpha$ be the connection component for ∇ with respect to local coordinates system z^1, z^2, \dots, z^n with respect to induced coordinate system $z^1, z^2, \dots, z^n, v^1, v^2, \dots, v^n$ of TM_{2n} . we set

$$\Gamma_{\beta\gamma}^\alpha = \Gamma_{\beta\gamma}^{\alpha \sim \alpha \sim} \quad \Gamma_{\beta\gamma} = 0$$

$$\tilde{\Gamma}_{\beta\gamma}^{\alpha \sim} = 0, \tilde{\Gamma}_{\beta\gamma}^{\alpha \sim \sim} = 0$$

$$\Gamma_{\beta\gamma}^\alpha = \frac{\partial \Gamma_{\beta\gamma}^\alpha}{\partial z^i} v^i, \Gamma_{\beta\gamma}^{\alpha \sim} = \Gamma_{\beta\gamma}^\alpha,$$

$$\tilde{\Gamma}_{\beta\gamma}^{\alpha \sim} = \Gamma_{\beta\gamma}^\alpha, \tilde{\Gamma}_{\beta\gamma}^{\alpha \sim \sim} = 0$$

Where the indices with \sim refer to v^1, v^2, \dots, v^n , then $\tilde{\Gamma}$'s are connection component of ∇^C .

Proposition 2.1: If T and R are the torsion and curvature tensor fields of affine connection ∇ , then T^C and R^C are respectively the torsion and curvature tensor fields of ∇^C .

Proof: proposition follows from the following formulae

$$T^C(Z^C, U^C) = (T(Z, U))^C = (\nabla_{z^U} - \nabla_{U^z} - [Z, U])^C$$

$$= \nabla_{z^C}^C U^C - \nabla_{U^C}^C Z^C - [Z^C, U^C]$$

$$R^C(Z^C, U^C) V^C = (R(Z, U) V)^C ([\nabla_z, \nabla_U]^V - \nabla[Z, U]^V)^C$$

$$= [\nabla_{z^C}^C \cdot \nabla_{z^C}^C] V^C - \nabla_{[Z^C, U^C]}^C V^C$$

Proposition 2.2: For any tensor field S and any vector field Z on complex manifold

M_{2n} We have

$$(i) \nabla_{z^C}^C (S^C) = (\nabla_z S)^C$$

$$(ii) \nabla^C (S^C) = (\nabla S)^C$$

$$(iii) \nabla_{z^C}^C (S^V) = (\nabla_z S)^V$$

$$(iv) \nabla^C (S^V) = (\nabla S^V) \quad = (\nabla_z S)^C$$

$$(v) \nabla_{z^V}^C (S^C) = (\nabla_z S)^V \quad = (\xi_z \nabla S)^C$$

$$(vi) \nabla_{z^V}^C (S^V) = (\nabla_z S)^V \quad = \xi_{z^C} (\nabla S)^C$$

Proof: As usual the suffices to verify these formulae in the special cases, where

$$S = f \in \mathfrak{S}_{0 \quad 2n}^0 M, S = df \in \mathfrak{S}_{1 \quad 2n}^0 M \text{ and } S = U \in \mathfrak{S}_{0 \quad 2n}^1 M$$

$$(i) \text{ If } S = f, \quad \text{then} \\ \nabla_{z^C}^C (f^C) = z^C f^C = L_{z^C} f^C = (L_z f)^C = (\nabla_z f)^C$$

If $S = U$, then the formula to be verified is

nothing but the definition of ∇^C . If $S = df$ or

more generally $S = W \in \mathfrak{S}_{1 \quad 2n}^0 M$, then

$$(\nabla_{z^V}^C W^C)(U^C) = \nabla_{z^C}^C (W^C (U^C)) - W^C (\nabla_{z^C}^C U^C)$$

$$= \nabla_{z^C}^C (W(U))^C - W^C (\nabla_z U)^C$$

$$= (\nabla_z W(U))^C - W^C ((\nabla_z U))^C$$

$$= (\nabla_z W(U))^C = (\nabla_z W)^C (U^C)$$

$$\text{Hence} \quad (\nabla_{z^V}^C W^C) = (\nabla_z W)^C$$

(ii) From (i), we have

$$\xi_{z^C} (\nabla^C S^C) = \nabla_{z^C}^C (S^C)$$

and hence

$$\nabla^C S^C = (\nabla S)^C$$

(iii) If $S = f$, then, we have

$$\nabla_{z^C}^C (f^V) = L_{z^C} (f^V)$$

$$= (L_z f)^V$$

$$= (\nabla_z f)^V$$

If $S = V$, then we write $Z = \xi^\alpha \frac{\partial}{\partial z^\alpha}$ and

$V = \eta^\alpha \frac{\partial}{\partial z^\alpha}$, then

$$\nabla_{z^C}^C (U^V) = \left(\xi^\alpha \frac{\partial \eta^\alpha}{\partial z^\alpha} + \Gamma_{\beta\gamma}^\alpha \xi^\beta \eta^\gamma \right) \frac{\partial}{\partial v^\alpha}$$

$$= \left(\xi^\alpha \frac{\partial \eta^\alpha}{\partial z^\alpha} + \Gamma_{\beta\gamma}^\alpha \xi^\beta \eta^\gamma \right) \frac{\partial}{\partial v^\alpha}$$

$$= (\nabla_z U)^V$$

Either by a similar calculation using a local co-ordinate, we get

$$\nabla_{z^C}^C (W^V) = (\nabla_z U)^V$$

For every $W \in \mathfrak{S}_1^0 M_{2n}$

- (iv) The proof is similar that of 2(ii).
- (v) This may be proved in the same way as 2(i)

$$\begin{aligned}\nabla_{z^C}^C (S^C) &= \nabla_{z^V} (\nabla^C S^C) \xi_{z^V} (\nabla S)^C \\ &= (\xi_z \nabla S)^V = (\nabla_z S)^V\end{aligned}$$

- (vi) The proof is similar that of 2(iii).

References

- [1] Yano, K. and S. Kobayashi: Prolongation of tensor fields and connections to tangent bundles, I general theory J. of mathematical .Japan 1 (1996) 199-210
- [2] Yano, K.: Differential Geometry on Complex and almost complex Spaces, Pregamon Press.
- [4] Sato I., Almost analytic vector field in almost complex manifold, Tohoku Math. J. 17(1965), 185-199.
- [6] Steenrod, N.: The topology of fiber bundles, Princeton Univ., Press, N.J. (1951).
- [7] Yano, K.: The theory of Lie derivatives and application, Amsterdam,(1957).
- [] Yano, K. and S. Ishihara: Tangent and Cotangent bundles, Differential Geometry, Marcel Dekker, New York, (1973).
- [8] Yano, K.: Tensor fields and connections on cross-section in the cotangent bundle, Tahoka Math. Jour. 19, (1967)32-48.