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Mathematical Modeling of Social Networks: A Graph Theoretical Approach

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Abstract
Social networks play a vital role in shaping information dissemination and interaction dynamics in modern societies. Understanding the structural properties of such networks is essential for identifying influential individuals and optimizing communication strategies. This study presents a graph-theoretical framework for modeling and analyzing social networks using a real-life–inspired university social system. The network is represented as an undirected graph, where nodes correspond to individuals and edges represent academic interactions. To address privacy and ethical concerns, synthetic data is employed to simulate realistic interaction patterns. Classical graph-theoretical measures, including degree, betweenness, and closeness centrality, are utilized to evaluate node importance and influence. The analysis reveals that nodes with high centrality values significantly enhance information dissemination efficiency within the network. The proposed approach demonstrates the effectiveness of graph theory as a mathematical tool for analyzing social structures and provides a transparent, application-oriented methodology suitable for both research and educational purposes. The study further highlights the potential for extending the framework to dynamic, weighted, and uncertainty-based graph models.

Keywords: *Social Networks, Graph Theory, Centrality Measures, Information Diffusion, Mathematical Modeling, Synthetic Data*

1. Introduction

The rapid growth of social networks has transformed the way individuals communicate, collaborate, and share information across diverse domains such as education, healthcare, business, and governance. Social networks, whether physical or virtual, consist of interconnected entities whose interactions collectively influence information dissemination, decision-making, and behavioral dynamics. Understanding the structural and functional properties of such networks has therefore become a critical research challenge.

Mathematical modeling offers a systematic framework for representing and analyzing complex social interactions. Among various modeling techniques, graph theory has emerged as one of the most powerful and intuitive tools for social network analysis. In graph-theoretical models, individuals are represented as vertices, while their interactions are captured as edges. This abstraction enables the use of rigorous mathematical concepts to quantify influence, connectivity, and information flow within a network.

The study of social networks using graph theory has expanded significantly in recent years, with research focusing on centrality measures, information diffusion patterns, community detection, and advanced analytical frameworks. Centrality measures remain foundational in the quantitative analysis of social network structures. Bendahman and Lotfi (2024) investigate various centrality metrics in complex networks, highlighting their effectiveness for identifying influential actors within social systems. Their work emphasizes how traditional centrality measures such as degree, betweenness, and closeness can be applied in graph models to optimize information flow and understand node importance.

Information diffusion processes in social networks have been extensively surveyed to understand how knowledge spreads across connections. Kesharwani and Ghosh (2024) provide a comprehensive taxonomy of information diffusion mechanisms in online social networks, detailing representative methods and approaches used to model and analyze spreading patterns. This literature underscores the role of network topology and structural properties in shaping diffusion outcomes. Recent research also examines the interplay between social network structure and narrative spreading. Gurung and Agarwal (2025) offer a survey on narrative diffusion, exploring how stories and narrative content propagate through social connections by considering a range of graph attributes and network dynamics. This work demonstrates the evolving interest in understanding content-specific diffusion beyond simple connectivity.

Community detection continues to be another major research theme in graph-based social network analysis. A systematic mapping study by researchers in Knowledge and Information Systems (2024) reviews machine learning-based techniques for identifying communities within networks. The study highlights overlapping and non-overlapping community structures and the range of algorithms applied to detect them, underscoring the integration of traditional graph theory with newer computational methods. A different perspective on social network analysis emphasizes enriched graph representations. A 2025 study in the International Journal of Computational Intelligence Systems proposes advanced community detection paradigms that integrate intrinsic communication patterns into graph models, aiming to enhance partition quality compared to classical algorithms. This approach reflects the trend of incorporating additional network attributes into graph representations to capture richer interaction dynamics.

Other literature highlights extensions of classical graph concepts to accommodate uncertainty and imprecise social relationships. Research on fuzzy social networks proposes extensions of centrality indices to fuzzy values, preserving the fluid nature of real social ties and providing a more nuanced analytical framework. Although this area is emerging, it points toward the increasing complexity with which modern social networks are modeled. Collectively, these studies demonstrate the breadth of recent research in graph-based social network analysis. They emphasize not only the continued relevance of traditional centrality and diffusion models but also the integration of machine learning, enriched representations, stochastic dynamics, and behavior-specific analyses to gain deeper insight into how social structures influence information flow and community formation.

Existing studies on social network analysis largely focus on large-scale online platforms such as Twitter, Facebook, and Instagram, emphasizing empirical data-driven approaches. However, relatively limited attention has been given to small and medium-scale real-world social systems, such as academic institutions or organizational networks, where interaction dynamics differ significantly. Moreover, many studies rely heavily on real datasets, which often pose challenges related to privacy, accessibility, and ethical considerations. There is a noticeable research gap in the development of illustrative, mathematically transparent case studies that demonstrate graph-theoretical analysis using synthetic yet realistic data.

Motivated by this gap, the present study proposes a graph-theoretical framework to model and analyze a university social network for efficient information dissemination. By employing synthetic data that mirrors real academic interactions, the study avoids privacy concerns while maintaining structural realism. Classical graph-theoretical measures such as degree, betweenness, and closeness centrality are used to identify influential individuals within the network.

The novelty of this work lies in its step-by-step mathematical modeling approach, combined with an application-oriented case study that bridges theory and practice. Unlike purely empirical analyses, this study emphasizes interpretability and methodological clarity, making it suitable for both academic research and pedagogical purposes.

The key objectives of this study are:

1. To model a real-life–inspired social network using graph theory.
2. To analyze network structure using centrality measures.
3. To identify influential nodes responsible for effective information dissemination.
4. To demonstrate the applicability of graph-theoretical concepts in real-world social systems.

Through this approach, the study highlights the significance of graph theory as a robust mathematical tool for understanding and optimizing social network dynamics.

2. Preliminary Concepts

This section outlines the fundamental mathematical and theoretical concepts required to understand the graph-theoretical modeling of social networks used in the present case study.

1. Social Network

A social network is a structured system of social entities (individuals or organizations) connected through relationships such as communication, collaboration, or influence. In mathematical modeling, social networks are represented using nodes and links, allowing quantitative analysis of interaction patterns.

2. Graph Theory

Graph theory is a branch of discrete mathematics concerned with the study of graphs, which are mathematical structures used to model pairwise relationships.

A graph is defined as:

G = (V, E)

where:

* (V) is a finite set of vertices (nodes)
* (E) is a set of edges (links) connecting pairs of vertices

3. Types of Graphs in Social Network Modeling

* Undirected Graph: Edges have no direction, representing mutual interactions.
* Directed Graph: Edges have directions, representing one-way influence.
* Weighted Graph: Edges have weights indicating interaction strength.
* Simple Graph: No self-loops or multiple edges.

In this study, the social network is modeled as an undirected simple graph.

4. Adjacency Matrix

The adjacency matrix ($A = [a_{\{ij\}}]$) represents node connectivity:

$$a_{\{ij\}} = \begin{cases} 1 & if\ an\ edge\ exists\ between\ v_i\ and\ v_j \\ 0 & otherwise \end{cases}$$

This matrix form enables computational analysis of large networks.

5. Degree of a Node

The degree of a node is the number of edges incident to it:

$$deg(v) = \sum_{\{j\}} a_{\{ij\}}$$

Nodes with high degree are considered locally influential.

6. Centrality Measures

Centrality measures quantify the importance of nodes in a network.

(a) Degree Centrality

$$C_{D(v)} = \frac{deg(v)}{\{n-1\}}$$

(b) Betweenness Centrality

Measures how often a node lies on shortest paths:

$$C_{B(v)} = \sum_{\{s \neq v \neq t\}} \frac{\{\sigma_{\{st\}}(v)\}}{\{\sigma_{\{st\}}\}}$$

(c) Closeness Centrality

Measures how close a node is to all others:

$$C_{C(v)} = \frac{1}{\{\sum d(v,u)\}}$$

7. Path and Shortest Path

* A path is a sequence of vertices connected by edges.
* The shortest path is the path with minimum length between two nodes.

Shortest paths play a vital role in information diffusion.

8. Information Diffusion in Networks

Information diffusion refers to how information spreads across a network through connected nodes. Nodes with higher centrality tend to disseminate information faster and more efficiently.

9. Network Density

Network density measures how connected a network is:

$$D = \frac{\{2|E|\}}{\{|V|(|V|-1)\}}$$

A higher density indicates stronger connectivity.

10. Community Structure

A community is a group of nodes with dense internal connections and sparse external connections. Community detection helps understand subgroup interactions in social networks.

These preliminary concepts establish the mathematical foundation required for modeling and analyzing social networks using graph theory, enabling the identification of influential nodes and understanding information diffusion mechanisms.

3. Methodology

This study adopts a graph-theoretical framework to model and analyze a real-life–inspired university social network for efficient information dissemination. Synthetic data is used to ensure privacy while preserving realistic interaction patterns. The methodology consists of the following systematic steps:

Step 1: Problem Identification and Network Definition
The primary objective is to identify influential individuals in a university social network who can effectively disseminate academic information, such as conference announcements.
The social network is defined as:
* Actors: Faculty, research scholars, students, and administrative staff
* Interactions: Academic collaboration, mentoring, and communication

Step 2: Data Generation (Synthetic Data Construction)
Since real social network data involves privacy concerns, synthetic data is generated based on realistic academic interaction patterns.
* Nodes represent individuals within the university
* Edges represent frequent academic or professional interactions
* The network is assumed to be undirected, implying mutual information exchange
This results in a finite node set (V) and edge set (E).

Step 3: Graph Modeling
The social network is mathematically modeled as an undirected graph:
G = (V, E)
where:
* ($V = \{v_1, v_2, \ldots, v_n\}$) denotes individuals
* (E = $\{(v_i, v_j)\}$) denotes interactions
Graph construction is performed using adjacency relationships derived from the synthetic dataset.

Step 4: Network Visualization
The constructed graph is visualized using a force-directed layout to:
* Identify structural patterns
* Observe hubs and bridge nodes
* Understand community formations
Visualization assists in preliminary qualitative analysis before mathematical evaluation.

Step 5: Centrality Measure Computation
To quantify node importance, classical graph-theoretical centrality measures are computed:
Degree Centrality-Measures direct connectivity:

$$C_D(v) = \frac{\{deg(v)\}}{\{n - 1\}}$$

Betweenness Centrality-Measures control over information flow:

$$C_B(v) = \sum_{\{s \neq v \neq t\}} \frac{\{\sigma_{\{st\}}(v)\}}{\{\sigma_{\{st\}}\}}$$

Closeness Centrality - Measures reachability speed:

$$C_C(v) = \frac{1}{\{\sum \ d(v,u)\}}$$

Step 6: Influential Node Identification
Nodes with:
* High degree
* High betweenness
* High closeness
are classified as influential nodes. These nodes are optimal candidates for initiating information dissemination.

Step 7: Information Diffusion Simulation
A simplified diffusion process is simulated:
* Information originates from a selected node
* It propagates through adjacent nodes at each discrete time step
* Coverage and speed of information spread are recorded
Different source nodes are compared to evaluate dissemination efficiency.

Step 8: Result Interpretation and Validation
Results are interpreted using:
* Graph metrics
* Structural patterns
* Diffusion outcomes
The effectiveness of the model is validated by comparing information spread initiated from central vs. peripheral nodes.

Step 9: Methodological Generalization
The proposed methodology is generalizable and can be extended to:
* Weighted and directed networks
* Dynamic social networks
* Fuzzy and neutrosophic graph models
* Real-world social media datasets

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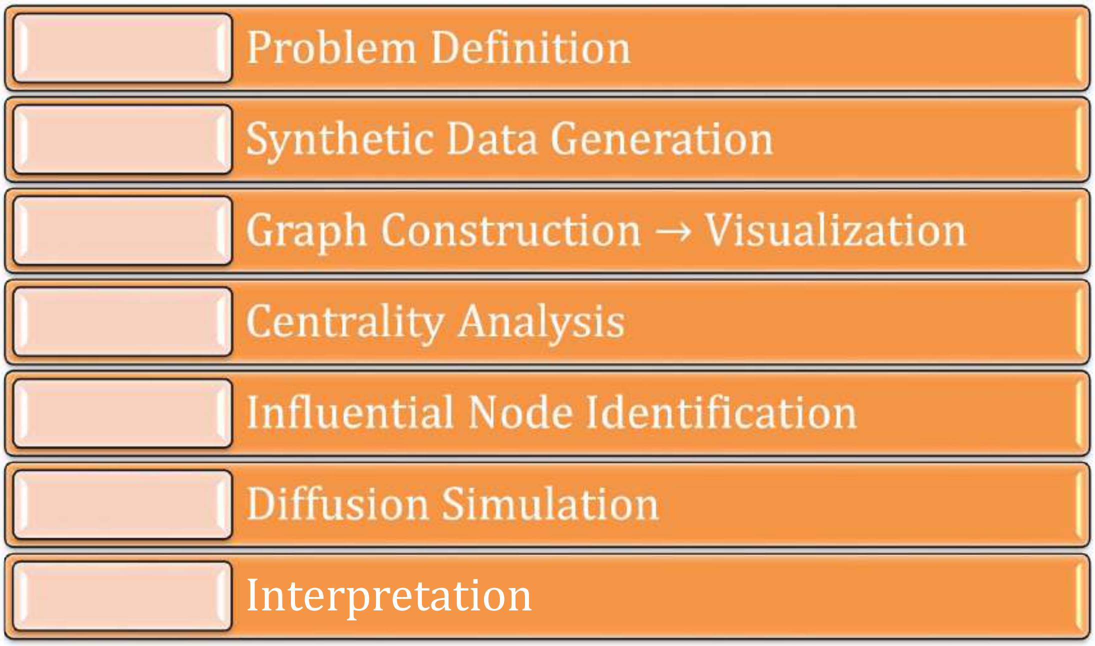


Methodology Flow

4. Case Study: Graph Theoretical Modeling of a University Social Network

Case Study Background

This case study considers a real-life–inspired academic social network within a university environment, where information related to an upcoming academic conference must be disseminated efficiently among students, researchers, faculty members, and administrative staff. The interactions among individuals form a complex network that can be mathematically modeled using graph theory.

Due to privacy and ethical concerns associated with real social media data, synthetic but realistic data is used to simulate social interactions within the campus.

Case Study Scenario

The university plans to circulate information regarding:

* Call for papers
* Registration deadlines
* Workshop schedules

The challenge is to identify influential individuals who can spread information rapidly across the academic community.

Graph-Based Network Representation

The social network is modeled as an undirected graph:

G = (V, E)

where:

* (V) represents individuals in the university
* (E) represents frequent academic interactions

Node Description (Actors)

Node ID	Description	Node ID	Description
N1	Senior Faculty	N6	Undergraduate Student – I Year
N2	Junior Faculty	N7	Undergraduate Student – II Year
N3	PhD Research Scholar	N8	Undergraduate Student – III Year
N4	Postgraduate Student I year	N9	Research Assistant
N5	Postgraduate Student II year	N10	Administrative Staff

Each node represents an active participant in academic communication through meetings, collaborations, or informal discussions.

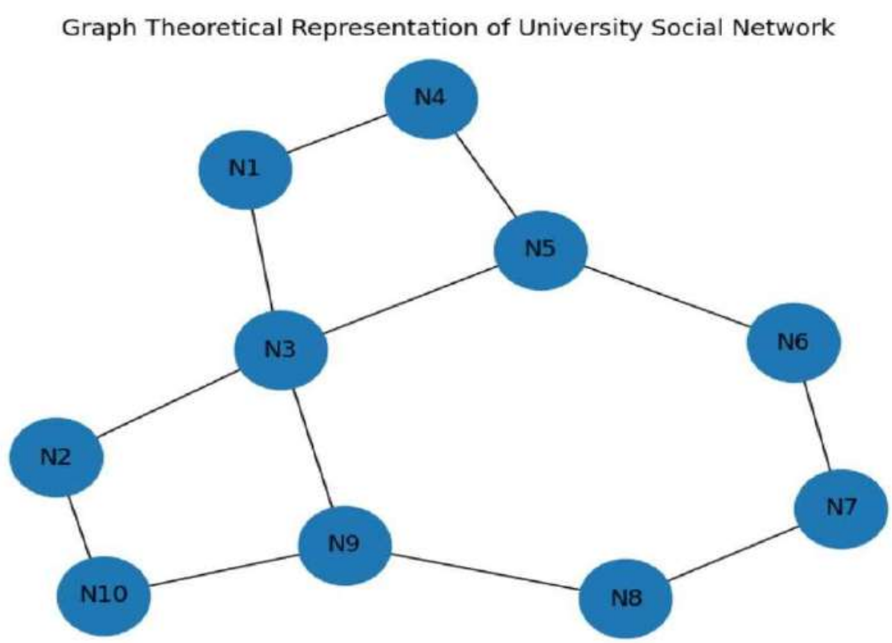
Edges denote regular academic interactions such as research collaboration, mentoring, or project discussions.

Edge Description (Interactions)

Edge	Connected Nodes	Edge	Connected Nodes
E1	N1 – N3	E7	N6 – N7
E2	N1 – N4	E8	N7 – N8
E3	N2 – N3	E9	N8 – N9

E4	N3 – N5	E10	N3 – N9
E5	N4 – N5	E11	N9 – N10
E6	N5 – N6	E12	N2 – N10

Centrality-Based Analysis
To identify influential members, classical graph-theoretic centrality measures are applied.
Degree Centrality
Nodes N3 and N5 exhibit higher degrees, indicating strong local connectivity and frequent interactions.
Betweenness Centrality
Nodes N3 and N9 show high betweenness values, acting as bridges between student groups and administrative staff.
Closeness Centrality
Node N3 has the shortest average path length to all other nodes, enabling rapid information dissemination.
Information Dissemination Observation
A simulated diffusion process assumes that information propagates through direct interactions at discrete time steps.
* When the conference information originates from N3, it reaches a majority of the network within a few interaction cycles.
* Initiating the information from peripheral nodes results in slower and limited spread.



Graphical Explanation of the Case Study Network
The figure represents the university social network modeled using graph theory, where:
* Vertices (Nodes) → Individuals in the university
* Edges (Links) → Academic or professional interactions
Mathematically, the network is an undirected graph G = (V, E)
1. Interpretation of Nodes
Each node corresponds to a specific role in the academic ecosystem:
* N1, N2 → Faculty members
* N3 → PhD research scholar
* N4, N5 → Postgraduate students
* N6, N7, N8 → Undergraduate students
* N9 → Research assistant
* N10 → Administrative staff
2. Interpretation of Edges
An edge between two nodes indicates frequent academic interaction, such as:
* Research collaboration
* Mentoring
* Project coordination
* Administrative communication
Since the graph is undirected, information exchange is bidirectional.
3. Structural Properties Visible from the Graph
(a) Hub Node Identification
* Node N3 is visually central and has multiple connections.
* It connects faculty (N1, N2), students (N5), and administration-related node (N9).
Graph-theoretic meaning:

N3 has high degree, high closeness, and high betweenness centrality.
(b) Bridge Nodes
* Node N9 acts as a bridge between:
  * Academic group (N3)
  * Administrative staff (N10)
  * Undergraduate chain (N8)
Meaning:
Removal of N9 would significantly disrupt information flow → high betweenness centrality. (c) Community Structure
* A student sub-network is visible:

$$N5 \rightarrow N6 \rightarrow N7 \rightarrow N8$$

 * Faculty and administration form smaller but influential clusters.
Meaning:
The network exhibits community structure, common in real social systems.
4. Information Flow Explanation Using the Graph
If information (conference announcement) originates at:
Node N3
* Information quickly reaches:
  * Faculty (N1, N2)
  * PG & UG students (N5 → N6 → N7 → N8)
  * Administration (via N9 → N10)
Fast, wide dissemination
Peripheral Node (e.g., N7)
* Information spreads slowly
* May not reach faculty or administration efficiently
 Limited dissemination
5. Graph-Theoretical Insight
From the graphical structure:
* The network resembles a small-world network
* Central nodes dramatically improve communication efficiency
* Graph theory helps identify optimal information broadcasters
Case Study Outcome
The analysis reveals that research scholars and central academic collaborators play a crucial role in disseminating information within university social networks. Graph theory provides a mathematically sound framework to identify such influential actors and optimize communication strategies.

5. Conclusion

This study demonstrated the effectiveness of graph-theoretical modeling in analyzing and understanding the structural dynamics of social networks. By representing a real-life–inspired university social network as an undirected graph using synthetic data, the research provided a mathematically transparent framework for examining interaction patterns and information flow. Classical graph-theoretical measures such as degree, betweenness, and closeness centrality were employed to identify influential individuals within the network.

The results revealed that nodes exhibiting high centrality values play a critical role in accelerating information dissemination across the network. In particular, research scholars and central academic collaborators emerged as key influencers, acting as hubs and bridges that connect different subgroups. The information diffusion analysis further confirmed that initiating communication from such influential nodes significantly improves both the speed and reach of information spread.

The study addresses an important research gap by focusing on a small-to-medium scale real-world social system and by utilizing synthetic data to overcome privacy and accessibility challenges. The proposed methodology is simple, interpretable, and adaptable, making it suitable for academic institutions, organizational communication networks, and similar social structures.

Overall, this work highlights the significance of graph theory as a robust mathematical tool for modeling social networks and supporting data-driven decision-making in information dissemination strategies. The findings reinforce the value of mathematical abstraction in capturing complex social interactions and provide a strong foundation for future research involving dynamic, weighted, and uncertainty-based network models.

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