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The Non-ideal Op Amp Method for Feedback Circuit Analysis

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Abstract: The non-ideal operational amplifier (op amp) technique is offered as a fresh way for analyzing feedback circuits. The approach accurately identifies the feedback type and determines the feedback network loading to the input and output of the amplifier, two of the primary challenges of the two-port analysis. In order to circumvent the issue of feedback type determination, the suggested technique generalizes the traditional op amp theory by considering every amplifier as a voltage amplifier. No feedback loading determination is required since all calculations are done assuming an open circuit with no load. An approach to accurately calculating the output impedance in current feedback is also shown in the paper.

Keywords: terms like "loop gain," "output impedance," "return ratio," and "two-port analysis"

Introduction

In electronics and control systems, the idea of feedback is essential. The term "feedback" refers to a method in which some of the output is sent back into the system. Degenerative or negative feedback occurs when the feedback signal is 90 degrees out of phase with the input. The most notable advantage of negative feedback is the desensitization of the closed-loop gain. The widening of bandwidth, the suppression of noise, and the amelioration of harmonic distortion are further advantages. Because negative feedback alters both input and output impedances, it may be used to adjust the driving impedance at a given port. The possibility for instability is a drawback that must be addressed throughout the design phase.

Because the feedback network loads the open-loop amplifier, even a simple examination of a feedback amplifier requires a lot of effort. Furthermore, particularly at high frequencies, it is not necessarily safe to assume that the amplifier and the feedback network are unidirectional. Assuming a one-way amplifier and feedback circuit, most textbooks describe feedback theory using a two-port approach [1, 2]. Before analyzing the circuit, one needs establish if the input signals are added in series or shunt, and what kind of feedback is being used (voltage or current). The feedback network's burden on the input and output is then accounted for when designing the open-loop amplifier. Using the open-loop gain A and the feedback factor f , one may determine the closed-loop gain. Using the two-port model, we simplify the study of feedback amplifiers.

you may read more on the methods in [4]. Similarly, two-

port analysis is the backbone of Yeung's method [5]. It is not always a simple process to identify the sort of feedback and the loading caused by the feedback network. Furthermore, utilizing the two-port technique to calculate the output impedance in current feedback might lead to inaccurate findings.

Bode [6] created the notion of feedback based on the RR concept. Setting all other sources to zero, severing the connection between the controlled source and the circuit, and then driving the circuit at the break point with an independent source of equal strength and calculating the resulting output through the feedback loop is how you arrive at the RR for a controlled source. Rosenstark [7] further developed the method. The impedance at any port may be calculated using Blackman's formula, [8]. There's good reason why the return ratio method is seldom covered in academic literature. It takes practice to identify the most straightforward dependant source that leads to the desired outcome. Otherwise, it might be a time-consuming process with little to no useful information gained.

Signal-flow diagrams provide yet another method for analyzing feedback circuits. The approach may in theory be used to any feedback architecture; however, picking the parameters that stand in for the various lines of feedback is a bit of a crapshoot. A technique based on the RR and accurate modeling of the amplifier without feedback was suggested by Nicolic et al. [10]. The ensuing loop gain and RR formulae are similar. Based on the cut-insertion theorem, Pellegrini [11] created a novel theory of feedback. Davis [12] first proposed the driving point impedance technique, and Ochoa [13] refined it.

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[13]. None of the above-mentioned techniques is particularly suited to undergraduate teaching.

In this paper, a general method for feedback circuit analysis is proposed based on the concept of the non-ideal op amp. The method treats all amplifiers as voltage amplifiers; hence, there is no need to determine the type of output sampling and input summation. The open-loop voltage gain, as well as, the input and output resistances refer to the unloaded amplifier and there is no need to calculate the loading that results from the feedback network. For quickness of calculations closed-loop expressions can be inserted in a spreadsheet. The non-ideal op amp methodology produces exact results as it does not make the assumption of unidirectional signal flow. The new method has been used in the class along with the traditional two-port technique and has been well received by the students.

The structure of the paper is as follows: In Section II the theoretical background for the analysis of both inverting and non-inverting feedback circuits is presented. Section III presents a number of examples, classified according to feedback type. In Section IV, an expression for the output impedance in current feedback is derived to complement the theory laid out in Section II. The paper closes by highlighting all significant contributions made to the field.

1. The Non-ideal Op Amp Method

Feedback Amplifiers

A high open-loop gain A is generated on purpose in a feedback design, and then the necessary gain is attained by subtracting f from the output at the input. The arrows in Figure 1 only point in one way, indicating that information

$$s_o = As_E = A(s_i - s_f) = As_i - fAs_o$$

From Eq. (1) the closed-loop gain is readily derived.

cannot go from the input to the output through the feedback network. Voltages or currents may represent signals denoted as s_i , s_f , s , and so on. Subtracting the feedback signal s_f from the input results in an error signal s_e , which in turn directly correlates to the output.

In two-port analysis, the sampled signal from the output must be identified as a voltage or a current. Shunt sampling is one method, while series sampling is another. It is also important to determine whether the feedback signal is introduced as a voltage (series mixing) or a current (shunt mixing) at the input node. By considering all of the permutations, there are essentially four types of feedback amplifiers.

$AV_f = v_o/v_i$ for voltage amplifiers; $G_{mf} = i_o/v_i$ for transconductance amplifiers; $R_{mf} = v_o/i_i$ for transresistance amplifiers; and $I_{mf} = v_o/i_i$ for current amplifiers. Assume that $AIf = i_o/i_i$.

In contrast to current feedback, which raises R_o , voltage feedback reduces it. Summation in series raises R_i , whereas shunt summation lowers it.

Non-inverting Amplifier

In the following paragraphs, we shall derive formulas for the input and output ports' changed voltage gain and driving point resistances due to the presence of feedback. The internal resistance of the source and the load linked to the output will be disregarded for the sake of simplicity. All parameters are assumed to be frequency independent as well. Figure 2 depicts the setup for a non-inverting operational amplifier. A voltage regulated voltage source with gain A is used to represent an amplifier, where R_i is the input resistance and R_o is the output resistance.

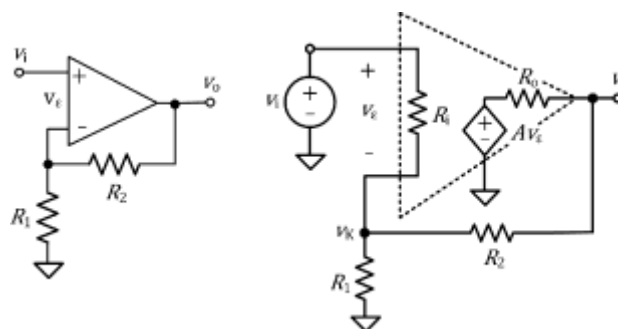


Figure 2. Left: Non-inverting op amp configuration with open-loop voltage gain A

Usually, the term $R_1 R_o$ in the numerator is much smaller than the other term. Dividing the numerator where the operator \parallel denotes the parallel combination of resistors and $f = R_1/(R_1 + R_o)$ the denominator cause a 1. To get some idea of their contribution of each term in a well-designed amplifier in the action of feedback, $f(A+1+R_o/R_i)$. Source internal resistance of an external load cause loop gain.

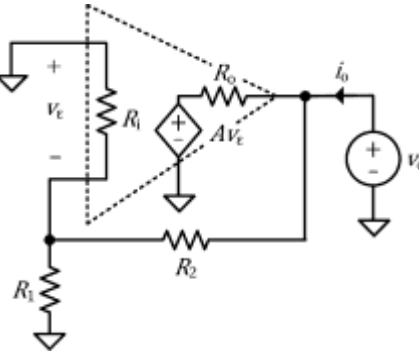


Table 1. Percentage Reduction in Open-loop Gain

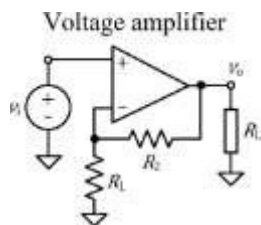
$\frac{R_2}{R_1 + R_2}$	$\frac{f f_o}{1 + A + \frac{R_o}{R_i}}$	$\frac{R_1 \parallel R_2}{R_i}$	$\frac{R_o}{R_1 + R_2}$
2.89%	96.75%	0.29%	0.07%

To find an expression for the closed-loop input resistance we express the ratio v_i/i_i as a function of the node voltages

and the denominator by $(R_1+R_2)R_i$ expression (4) is written in a more insightful form.

Figure 3. Equivalent circuit for the calculation of the closed-loop output resistance

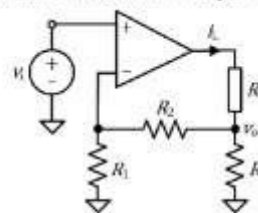
Among others, two basic feedback circuits can be implemented with the non-inverting topology: the voltage amplifier and the transconductance amplifier. Figure 4 gives the expression for the ideal gain A_∞ (obtained when $A \rightarrow \infty$), as well as, the relation of the closed-loop voltage gain A_{vf} to the desired closed-loop parameter.



$$A_{\infty} = 1 + \frac{R_2}{R_1}$$

$$\frac{v_o}{v_i} = A_{vf}$$

Transconductance amplifier



$$A_{\infty} = \frac{R + R_1 + R_2}{R R_1}$$

$$G_{mf} = \frac{i_o}{v_i} \approx \frac{A_{vf}}{R \parallel (R_1 + R_2)}$$

real gain and closed-loop parameter as a function of

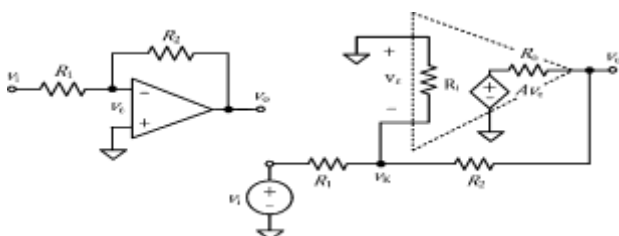
ing Amplifier

where D is taken from (5).

To derive an expression for the closed-loop output resistance we refer to the equivalent circuit of Figure 3, where the input has been connected to the ground and we have introduced the independent voltage source v_o that drives the amplifier output, producing a current i_o . Current

1 2

the gain reduction comes from the action of feedback, i.e. from the term $f(A+1+R_o/R_i)$.



The inverting op amp configuration is depicted in Figure 5, along with its equivalent circuit. The parameters have the same meaning as in the previous paragraph. The node equations are written as

Examples of Feedback Amplifiers

Analyzed With the New Methodology Series-shunt Feedback

In series-shunt feedback, the output voltage is sampled with a voltage divider and the feedback signal is subtracted in the input loop. The amplifier of Figure 7 is a typical case. Because of the asymmetry that exists in the differential stage inputs,

the amplifier does not comply with the simple model of Figure 1 and the application of the two-port method is inappropriate. Neglecting feedback resistors R_1, R_2 the unloaded open-loop gain, input and output resistances are as follows

Figure 5. Left: Inverting op amp configuration. Right: expressions for the closed-loop parameters indicated by the subscript “f”.

Solve (12) for v_K and substitute the result in (15) to get

$$D = \frac{R_1 r_o + 2A r_{\pi 1} (R_1 + R_2)}{(R_1 + 2r_{\pi 1})(R_2 + r_o) + 2(A+1)r_{\pi 1} R_1}$$

There is no need to calculate the output resistance, since the equivalent circuit is the same to that for the non-inverting amplifier (Figure 3). Expression (11) also holds for the inverting amplifier.

$$R_{of} = \frac{R_1 + R_2 + r_o}{\left[\frac{R_1 R_2 + 2r_{\pi 1} (R_1 + R_2)}{(R_1 + 2r_{\pi 1})(R_2 + r_o) + 2(A+1)r_{\pi 1} R_1} \right] r_o}$$

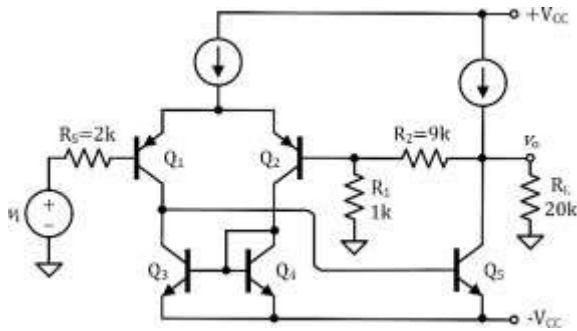


Figure 7. A two-stage voltage amplifier

Assuming $h_{fe} = 100$, $g_{m1} = 4$ mS, $r_{\pi 1} = 25$ k Ω , $g_{m5} = 40$ mS, $r_{\pi 5} = 2.5$ k Ω , $r_o = 50$ k Ω we find $A_{vf} = 9.97$, $R_{if} = 16.7$ M Ω , $R_{of} = 24.4$ Ω . While the input and output resistances are a property of the amplifier itself, the gain from the input to output should account for the source internal resistance

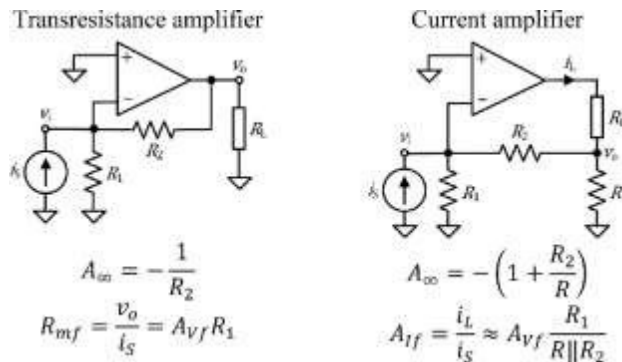


Figure 6. Ideal gain and closed-loop parameter as a function of the closed-loop voltage gain

controlled source

$$A = g_{m1} r_{\pi 5} g_{m5} r_o, R_i = 2r_{\pi 1}, R_o = r_o$$

where g_m is the transistor transconductance, r_{π} its input resistance and r_o the total load at the collector of Q_5 excluding R_L . Using equations (4), (8), (11) we find the

The inverting design is often used to build transresistance and current amplifiers. In Figure 6, we see the relationship between the ideal gain A and the non-ideal op amp-obtained voltage gain A_{vf} , as well as the formula for A itself.

With this, our examination of the two fundamental amplifier layouts is complete. Analyzing non-inverting amplifier circuits may begin with equations (4), (8), and (11), whereas inverting amplifiers need the application of equations (13), (16), and (11). In the following paragraphs, we will use the model with A derived from (17) to investigate many different types of feedback circuits. These are not rough estimates but rather explicit expressions. The signal path and amplifier's internal structure have not been assumed under any circumstances. Take note of the phrase that occurs in all of the formulations in (4), (8), and (11) that stem from the common term D . Since in this case A is the unloaded open-loop gain, this term differs from the $1+A$ amount in the two-port theory.

and also the external load connected to the output. Assuming $R_s = 2 \text{ k}\Omega$, $R_L = 20 \text{ k}\Omega$ the total gain is computed as

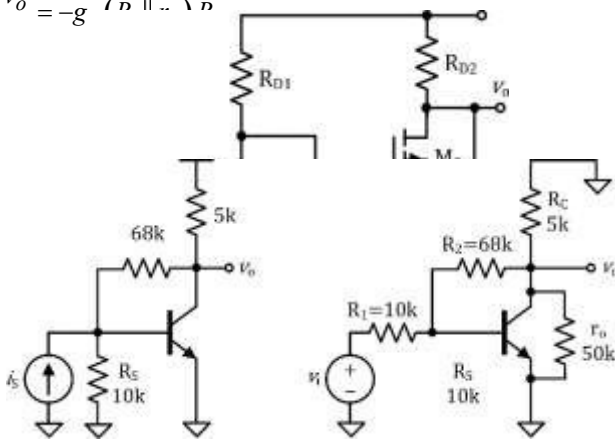
Shunt-shunt Feedback

Shunt-shunt feedback takes a reading of the output voltage and injects a current into the system that is proportional to the reading.

The SPICE simulation validates all of the findings.

Figure 6 in reference [14] is another illustration of a voltage amplifier. According to this reference, "this differs from the basic voltage-feedback structure in that the current flowing into the left-hand side of the feedback network is not the input current to the amplifier without feedback (which happens to be zero

$$A = \frac{v_o}{i_i} = -g_m R_{D1} R_{D2}$$



With reference to the non-ideal op amp method, when feedback is applied to a point of low resistance, R_o unchanged. In FETs the transformation factor is $\mu+1$ where $\mu = g_m r_d$, where r_d is a high value resistor, usually the resistance "seen" when looking at the After doing the calculations the value obtained for the closed-loop input resistance R_{if} should be multiplied by $\mu+1$ (or $h_{fe} + 1$).

Neglecting R_1 , R_2 the open-loop voltage gain and output resistance of the amplifier of Figure 8 are the closed-loop resistance at the base of the transistor after

$$A = g_{m1} R_{d1} g_{m2} R_{d2}, R_o = R_{d2}$$

closed-loop transresistance we write

From (4), (8) we get the expressions for the closed-loop

Shunt-series Feedback

Assuming $g_{m1} = g_{m2} = 20 \text{ mS}$, $R_{d1} = R_{d2} = 5 \text{ k}\Omega$, $R_1 = 1 \text{ k}\Omega$, $R_2 = 9 \text{ k}\Omega$ we get: $A_{vf} = 9.72$, $R_{if} = 686 \text{ G}\Omega$, circuit with the non-ideal op amp method we need to convert the current source to its Thevenin equivalent circuit. The output is the point where feedback is applied, therefore the open-loop voltage gain v_o/v_i ignoring R_1 , R_2 ($R_1 = 0$, $R_2 \rightarrow \infty$) is

The open-loop input resistance is $R_i = r_i = 10 \text{ k}\Omega$

$$A_{vf}$$



$R_{of} = 92.3 \text{ }\Omega$. SPICE simulation produces exactly the same results.

in the case of a FET); rather, it is the source (or drain) current of the first stage."

input node. The closed-loop parameter $R_{mf} = v_o/i_i$ is called transresistance or mutual resistance. To apply the non-ideal op amp method to the inverting amplifier of Figure 9 the current source i_s along with its internal resistance R_s should be converted to their Thevenin equivalent, see right side schematic where R_s has been renamed as R_1 . Then the open-loop parameters can be calculated by setting R_1 to zero and R_2 to infinity.

such as the emitter of a bipolar transistor or the source of a FET, a transformation of resistors is necessary, see [16]. For bipolars R_1 , R_2 , R_o should be multiplied by $h_{fe}+1$.

Figure 9. Single transistor transresistance amplifier

Using equations (13), (16), (11) the expressions for the closed-loop parameters are as follows

$$(R_1 + r_\pi)(R_2 + R') + r_\pi R_1 (1 + g_m R')$$

drain. For a value of 10^9 for r

subtracting R_1 . Assuming $g_m = 40 \text{ mS}$, $r_\pi = 2.5 \text{ k}\Omega$, $r_o = 50 \text{ k}\Omega$, $R_1 = 10 \text{ k}\Omega$, $R_2 = 68 \text{ k}\Omega$ we find $A_{vf} = -5.64$, $R_{if} = 342.5 \text{ }\Omega$, $R_{of} = 726 \text{ }\Omega$. The resistance that the current source sees is $342.5 \parallel 10,000 = 331 \text{ }\Omega$. To calculate the

Shunt-series feedback is especially suited to current amplifiers. In the amplifier depicted on the left side of

and the output resistance $R_o = R \parallel (R_L + r_o) = 93.75 \text{ }\Omega$. From equations (4), (8), (11) we obtain the closed-loop parameters: $A_{vf} = 12.56$, $R_{if} = 47 \text{ k}\Omega$, $R_{of} = 19 \text{ }\Omega$. Because of the series feedback applied to the output, the R_{of}

value is erroneous. The closed-loop transconductance is calculated as

Figure 10.

Left: current amplifier. Right: AC equivalent circuit with the current source transformed to a voltage

source

The open-loop output resistance is $R \parallel (1/g_m + R_L)$. Assuming $g_m = 0.02$ S, $r_o = 50$ k Ω , $R_L = 500$, $R = 4$ k Ω , $R_1 = 10$ k Ω , $R_2 = 20$ k Ω we find $A = -440$, $R_o = 483.5$ Ω .

For numeric computation purposes we will use $R_i = 10^9$ Ω . Using eqs. (13), (16) with $R_1 = 10$ k Ω , $R_2 = 20$ k Ω we get $A_{vf} = -1.986$, $R_{if} = 46.5$. The resistance that the current source sees is $46.5 \parallel 10,000 = 46.3$ Ω . By making the approximation that the AC voltage at the gate of M_1 is zero the closed-loop current gain is computed as

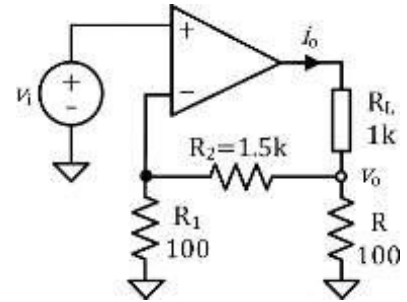


Figure 11. Transconductance amplifier drawn in op amp form

The aforementioned findings are supported by SPICE simulation, which estimates a load resistance of 591.8 k at the output. Our calculated output resistance is obviously off. We shall determine the real resistance that the load "feels" in the next subsection.

For the vast majority of real-world circuits, approximation using Equation (32) is enough. If one uses the formula to determine v_g , then one may determine the precise value of the present gain.

When in voltage mode, the amplifier's output

3.4. Series-series Feedback

The input loop. In the transconductance amplifier of

Figure 11 the amplifier is assumed to have voltage gain $G = 1000$, input resistance $r_i = 10$ k Ω and output resistance $r_o = 500$ Ω . As mentioned in a previous paragraph the output is the point where feedback is taken from. The open-loop voltage gain neglecting R_L ,

resistance is the value provided by relation (11), which is the resistance at the output node. The resistance that the load "feels" as a result of the feedback action is of relevance in series output feedback, also known as current feedback. The greater the resistance, the more stable the stream. Either the emitter or the collector of the output transistor may be used to connect to the load. To begin, let's look at the former scenario. In electronics, an analogous circuit is

R_2 is

$$R \parallel R_1 \parallel R_i + R_2 \quad \text{or} \quad A \quad R_1 \parallel R_i + R_2 + R$$

Substituting (38), (39) to (37) and then solving for v_o/v_i we obtain the expression for the output resistance in seriesfeedback.

$$\frac{[R_2 + (A+1)(R_1 \parallel R_i)]R}{R_2 + (A+1)(R_1 \parallel R_i)}$$

$$R_{emitter} = r_e + (G+1)R$$

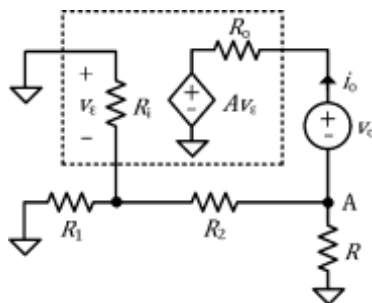
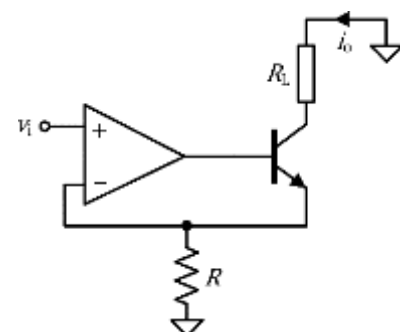


Figure 12. Equivalent circuit for the calculation of the output impedance when "looking" to the emitter

The resistance when looking down to the collector of the transistor (Figure 13) can be computed from the equation

$$R_{emitter} = r_e + (G+1)(R \parallel R_1) \text{ In our case } R_1 \rightarrow \infty \text{ and (43) simplifies to}$$



$$R_{oD,series} = (1 + g_m R_{oS,series}) r_o$$

Figure 13. The resistance when looking down to the collector of the transistor.

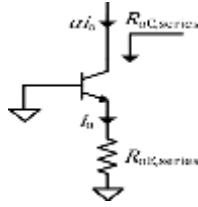


Figure 14. Calculation of the resistance presented to the load in a transconductance amplifier

Having derived equation (40) we are now able to calculate the output resistance “seen” by the load for the circuits of Figure 10 and Figure 11. For the circuit of Figure 10 (right side) the open-loop gain and output resistance are computed assuming infinite resistance connected at the source of transistor M_3 . As usual we neglect resistors R_1 , R_2 . It is found that $|A| = g_m r_o / 2 = 500$, $R_i \rightarrow \infty$, $R_o = 1/g_m = 50 \Omega$. Using (40) with $R = 4 \text{ k}\Omega$, $R_1 = 10 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$ we find $R_{oS,series} = 591.8 \text{ k}\Omega$ which is also the value predicted by the simulation.

For the circuit of Figure 11, setting $R_1 = 0$ and disconnecting the amplifier output from the load, we get: $A = 1000$, $R_i = 10 \text{ k}\Omega$, $R_o = 500 \Omega$. Then from eq. (40) with $R = 100 \Omega$, $R_1 = 100$, $R_2 = 1.5 \text{ k}\Omega$ we find $R_{of,series} = 6.42 \text{ k}\Omega$, a value that is verified by SPICE.

One last example to be examined is the transconductance amplifier of Figure 12 taken from [15]. In this work, it is mentioned that the two-port method fails to calculate the correct resistance at the collector of the transistor. For the amplifier depicted with the triangle we will assume a gain $G = 1000$, $r_i \rightarrow \infty$, $r_o = 0$. The bipolar transistor parameters are: $h_{fe} = 100$, $g_m = 40 \text{ mS}$, $r_\pi = 2.5 \text{ k}\Omega$. We will first calculate the resistance seen at the emitter. For this reason, the inverting input is connected to the ground and the emitter is connected to a load with high AC impedance,

Substitution of values gives $R_{collector} = 5.05 \text{ M}\Omega$ which is exactly the value predicted by SPICE simulation. If the gain G is large the resistance at the collector becomes $R_{collector} = (h_{fe} + 1)r_o$.

2. Discussion

An alternative to the standard two-port approach, which uses non-ideal operational amplifiers, has been presented. It gets around the two major issues with the two-port approach, which are type identification and input/output feedback loading determination. Since all amplifiers are assumed to be voltage amplifiers, the first problem disappears, and the second is solved by determining the open-loop numbers in their unloaded form. No assumptions are made about the construction of the amplifier or the signal flow, therefore the findings are precise. Equations may be entered into a spreadsheet

to get numerical results. This is both efficient and accurate. Maple, Matlab's Symbolic Toolbox, or SymPy (Python's Symbolic Mathematics Library) may be used to get the complete closed-loop expressions. The amount of work involved is low. The return ratio analysis and other approaches cannot do this.

The open-loop gain is loaded differently depending on the values of the parameters, and the non-ideal op amp approach gives some insight into this. In this approach, even novice and seasoned designers may easily determine which of their parameters might benefit from tweaking.

The suggested technique is described in terms of an open-loop gain and resistances that are kept constant at all times. But there's no reason it couldn't also operate with components or characteristics that rely on frequency. To this aim, impedances may stand in for resistors, and a frequency-dependent equation for the open-loop gain, $A(f)$, can be used instead.

Here are the measures taken to implement the suggested procedure:

- I. Determine if the amplifier inverts the input signal; II. Substitute the Thevenin equivalent circuit for any current source connected to the input; III.

Determine the unloaded voltage gain, input resistance, and output resistance by locating the feedback resistors R_1 , R_2 .

To restore the feedback to a low-resistance location, such the emitter of a bipolar transistor or the source of a field-effect transistor (FET), a resistance transformation is required.

Determine the closed-loop parameters (iv). Eqs. (4), (8), and (11) should be used in the non-inverting situation. Apply Eqs. (13), (16), and (11) for the inverse situation. the required closed-loop quantity may be expressed as a function of AV_f and the other circuit parameters if necessary (step v).

In undergraduate classes in analog circuits, the suggested technique has been utilized for a while as a tool for teaching feedback. Students found its implementation less complicated than other approaches. The non-ideal op amp is usually used to acquire the right answer fast, after which it is compared to more traditional approaches that may reveal additional light on the role of feedback

3. Conclusions

The non-ideal op amp technique suggested here is grounded on the established canon of op amp circuit theory. The approach is generic since it does not presume anything about the amplifier's internals or the path taken by the signal. According to the suggested technique, every amplifier is modeled as a voltage amplifier. Because only the unloaded amounts need to be determined, its implementation is straightforward. By doing so, the two primary problems plaguing the implementation of the two-port technique, namely, determining the kind of feedback and determining the loading from the feedback network, are resolved automatically.

To complement the non-ideal operational amplifier technique, a method has been provided that accurately calculates the output impedance during current feedback. Whether the load is connected to the emitter or the collector of the transistor, the two-port analysis cannot tell the difference in this scenario.

Although the paper's derived expressions all include resistors, the suggested approach may be applied to

circuits using components whose frequency changes. Transfer functions are being calculated using the non-ideal op amp approach.

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